

Location and reorganization problems: the Calabrian health care system case

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Abstract

In the last few years, the Italian healthcare system has been coping with radical changes, aimed at guaranteeing more efficiency while containing costs. Starting from the actual service network organization, we discuss the problem faced by the Italian authorities of reorganizing the healthcare service network and we propose some optimization models to support the decision-making process.

In the first part of the work, we compare the existing health care service network of the northern area of Calabria (Italy) with the configurations determined by solving well-known facility location models.

In the second part, taking into account the healthcare reorganization plans imposed by local governments, we consider the problem of reorganizing the public health care service network of the northern area of Calabria. Indeed, we propose two ad-hoc optimization models that consider national and regional guidelines and constraints. The behaviour of the proposed models, in terms of solution quality, is evaluated on the basis of an extensive computational study on real data.

Keywords: *Location models, health service network, reorganization, real data.*

1 Introduction

Facility location decisions play a crucial role at a strategic level, for both private and public sectors, as they have a strong impact on operational performances. In particular, in the public sector, the localization of services can deeply affect the accessibility of users and, hence, the quality of the service, this is the case of hospitals, schools and emergency services.

The Facility Location Problem (FLP) literature is quite extensive and several formulations have been proposed, taking into account different objective functions, features of the facilities, characteristics of the demand and its interaction with facilities.

Some interesting reviews of models and methods for addressing FLP are proposed in [1, 4, 3, 8, 23, 2, 6, 5]. An accurate identification of the parameters that can be used to classify FLP is presented in [3].

The three classes of facility location models mainly used in a public context are the *median*, *covering* and *center* models.

In the *median models* the objective is to find the optimal location of exactly p (p -median problems) facilities, so that the (weighted) sum of the distances between customers and their

assigned facilities is minimized. The first formulation of the p-median problem is described in [7], another seminal paper on the subject is [9].

Covering models are constructed by introducing the so called *coverage radius* and by considering a facility covering a demand point, if their distance does not exceed the predefined coverage radius. More in detail, there are two different formulations of covering models: the *Set Covering Location Problem* (see [10, 11, 12]) where the objective is to minimize the number of facilities to cover the entire demand in a given area and the *Maximum Covering Location Problem* (see [13, 14]), where the objective is to maximize the covered demand under a limited number of facilities.

These models have various real world application in the public sector, we cite for example the cases of locating emergency services (see [17]), bus stops (see [18]), health clinics (see [19]). Another interesting application is related to the hierarchical health services case, where the objective is to localize structures that can supply different types of services and that are related in a hierarchical manner ([20, 15, 16]). A good review of hierarchical facility models is also presented in [22].

In the *center models*, in a worst case setting, the objective is to minimize the maximum distance between demands and their allocated facilities. This class of problems involves locating one or more facilities in such a way that every demand receives its service from the closest facility and the maximum distance between each demand node and its facility is as small as possible. In fact, center location problems are usually used in locating emergency service, where the quickest response time represents the most important objective. The center models were first introduced by Hakimi in [7, 21], where the location of a single facility on a network is considered.

Some works related to facility location in healthcare applied to a real case are presented in [24, 25, 26]

In some practical applications, starting from an existing facility network, a reorganization, involving some changes to the set of facilities that are currently active in the area of interest, is required. The changes can be related to the number or the position of facilities, to their capacities and/or types of supplied services. In the literature, the reorganization follows on variations of some of the parameters that influence the current location, like demand, costs, market structure, degree of competition. Indeed, owing to the variability of such parameters, the current scenario could be inefficient and requires a reorganization of the system. The most variable parameter discussed in the literature is the demand. The reorganization can take place either before (ex-ante) the changes occur, on the basis of forecasting the future, or after the changes have happened (ex-post).

The ex-ante reorganization models can also be distinguished in deterministic multi-period models, where the decisions are made in different time periods of a given planning horizon, on the basis of the known value of parameters over time, and stochastic models, where decisions are made once on the basis of an uncertain knowledge of the future value of input parameters. Some important contributions related to multi-period models can be found in [27, 28, 29, 30, 31, 32, 33, 34, 35], while some interesting and recent stochastic models are presented in [36, 37].

The rationale of ex-post reorganization models is to change the actual facility layout, according to some new defined rules, by modifying either objective function or constraints or both. In [38] reorganization models are formulated in order to meet the new demand distribution. In [39] the authors consider the case where the reduction in the number of existing facilities is necessary for economic contingencies. In particular, they consider two cases where firms need to reduce their facilities, one related to firms operating in a competitive market and the other relative to a

market without competition. In the first case, firms aim at maximizing the retained demand (or minimizing demand loss to competitors) when the facilities are reduced by a specified number. In the second case, the firms need to close some facilities to reduce costs, with the objective of minimizing the uncovered population.

In this paper, we address localization and reorganization problems arising in the Italian health care system. In particular, attention is focused on the specific situation of a region in the South of Italy (i.e. Calabria). We first describe the current localization of public hospitals in Calabria; successively, through the definition of appropriate performance indexes, we compare the actual configuration with an ideal localization, obtained by applying some classical facility location models to real data.

The second part of the work is focused on reorganization models and it is dedicated to developing some optimization models, supporting the reorganization of the regional public hospitality network, owing to new governmental guidelines. In particular, we consider two different situations. In the first scenario, the aim is to minimize the number of public hospitals to be opened, considering the local authorities constraints, without taking explicitly into account the demands. The obtained configuration is then utilized as input for a covering model, in order to evaluate the quality of service of the solution in terms of satisfied demand. In the second scenario, the aim is to satisfy the demand in the new hospital configuration, by reallocating part of the demand to the retained facilities, whose capacity can be expanded within definite limits.

The rest of the paper is organized as follows: Section 2 presents some classical localization models that are applied to the scenarios under study; while in Section 3 we formulate two new optimization models to address the reorganization issues faced by local authorities in designing the current health care regional network; Section 4 compares, in a computational study, the actual health care facilities network with the configuration obtained from the application of the described models on a set of real data; the paper ends with Section 5, where we draw some conclusions.

2 Facility location models

In this section, we describe some classical facility location models that we will further apply to the problem under study.

The following parameters and variables are necessary to formulate the considered mathematical programming models.

- Parameters
 - $N = 1, \dots, n$ the set of possible hospital locations;
 - $R = 1, \dots, r$ the set of possible hospital departments. It is worth noting that each hospitals can contains only a sub-set of departments;
 - $M = 1, \dots, m$ the set of demand centers, i.e. the potential departments users;
 - l_j^k the capacity in terms of number of beds of department k , $k = 1, \dots, r$ in hospital j , $j = 1, \dots, n$;
 - d_i^k the demand from center i , $i = 1, \dots, m$ to department k $k = 1, \dots, r$.
- Variables

- y_j binary variable equal to 1 if hospital j is opened, 0 otherwise;
- x_{ij}^k the fraction of demand from center i served by department k of hospital j ;
- c_{ij} the connection cost expressed in terms of travel time between the center i and the hospital location j ;
- t_{ij} the distance between the center i and the hospital location j .

The first formulation is a classical *p-median* problem (for more details, the reader is referred to [9]). The aim is to locate p hospitals, while assigning the demand from the centers to departments, with the objective of minimizing the total network travel time.

The mathematical formulation of the p-Median Hospitals Location Problem (pMedian_HLP, for short) is reported in what follows.

$$\min \quad \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^r c_{ij} x_{ij}^k \quad (1)$$

$$\sum_{j=1}^n x_{ij}^k = 1, \quad \forall i = 1, \dots, m, \quad k = 1, \dots, r \quad (2)$$

$$\sum_{j=1}^n y_j = (\leq) p \quad (3)$$

$$\sum_{i=1}^m d_i^k x_{ij}^k \leq l_j^k y_j \quad \forall j = 1, \dots, n, \quad k = 1, \dots, r \quad (4)$$

$$x_{ij}^k \geq 0, \quad \forall i = 1, \dots, m, \quad j = 1, \dots, n, \quad k = 1, \dots, r \quad (5)$$

$$y_j \in (0, 1), \quad \forall j = 1, \dots, n \quad (6)$$

The objective (1), to be minimized, is the total travel time between hospitals and demand centers. Constraints (2) ensure that all demand coming from center i will be satisfied. Constraints (3) guarantee the localization of exactly p (at most p) hospitals; constraints (4) impose that the overall demand, corresponding to a given hospital department, does not exceed the department capacity. Constraints (5) and (6) define the variables domain.

The problem under consideration has been also addressed by considering the *p-center* model [7],[21]. The objective is to minimize the maximum distance between each demand center and its nearest hospital.

Let T denote the maximum distance between each demand center i and the hospital where i is assigned, the pCenter_HLP model assumes the following form:

$$\min \quad T \quad (7)$$

$$\sum_{j=1}^n x_{ij}^k = 1, \quad \forall i = 1, \dots, m, \quad k = 1, \dots, r \quad (8)$$

$$\sum_{j=1}^n y_j = (\leq) p \quad (9)$$

$$T - \sum_{j=1}^n d_i^k t_{ij} x_{ij}^k \geq 0 \quad \forall i = 1, \dots, m, \quad k = 1, \dots, r \quad (10)$$

$$\sum_{i=1}^m d_i^k x_{ij}^k \leq l_j^k y_j \quad \forall j = 1, \dots, n, \quad k = 1, \dots, r \quad (11)$$

$$x_{ij}^k \geq 0, \quad \forall i = 1, \dots, m, \quad j = 1, \dots, n, \quad k = 1, \dots, r \quad (12)$$

$$y_j \in (0, 1), \quad \forall j = 1, \dots, n \quad (13)$$

Another version of the problem under study, where the aim is to minimize the number of hospitals to be activated, while respecting capacity constraints and ensuring the coverage of all the demand (referred as Covering_HLP1) is given in the following:

$$\min \quad \sum_{j=1}^n y_j \quad (14)$$

$$\sum_{j=1}^n x_{ij}^k = 1, \quad \forall i = 1, \dots, m, \quad k = 1, \dots, r \quad (15)$$

$$\sum_{i=1}^m d_i^k x_{ij}^k \leq l_j^k y_j \quad \forall j = 1, \dots, n, \quad k = 1, \dots, r \quad (16)$$

$$x_{ij}^k \geq 0, \quad \forall i = 1, \dots, m, \quad j = 1, \dots, n, \quad k = 1, \dots, r \quad (17)$$

$$y_j \in (0, 1), \quad \forall j = 1, \dots, n \quad (18)$$

The last formulation (referred to as Covering_HLP2) is mathematically represented by using a *covering* model [1], where the objective is to assign to an hospital the demand centers located at no more than a known coverage-distance from the hospital.

Let S be the coverage-distance and let a_{ij} be a parameter equal to 1 if the distance from i to j is less than S and 0 otherwise, the Covering_HLP2 can be stated as follows:

$$\max \quad \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^r a_{ij} d_i^k x_{ij}^k \quad (19)$$

$$\sum_{j=1}^n y_j = (\leq) p \quad (20)$$

$$x_{ij}^k \leq a_{ij} \quad \forall i = 1, \dots, m, \quad j = 1, \dots, n, \quad k = 1, \dots, r \quad (21)$$

$$\sum_{i=1}^m d_i^k x_{ij}^k \leq l_j^k y_j \quad \forall j = 1, \dots, n, \quad k = 1, \dots, r \quad (22)$$

$$\sum_{j=1}^n x_{ij}^k \leq 1, \quad \forall i = 1, \dots, m, \quad k = 1, \dots, r \quad (23)$$

$$x_{ij}^k \geq 0, \quad \forall i = 1, \dots, m, \quad j = 1, \dots, n, \quad k = 1, \dots, r \quad (24)$$

$$y_j \in (0, 1), \quad \forall j = 1, \dots, n \quad (25)$$

3 The health care network reorganization problem (HNRP)

The health care system reorganization process in Italy began in 2004 with a law (Art. 1 comma 180 of the law 13 December 2004 number 311) that forces Regional Authorities to re-qualify and reorganize the health regional systems every three years.

The Calabria Region (South of Italy) Government, with DGR number 585 of 10/09/2009, defined some guidelines in order to start the process that, during the next few years, will lead to the reorganization of the Calabrian health care network.

On the basis of these guidelines, the responsible authorities, that is the “Azienda Sanitaria Provinciale (ASP)”, acts to reorganize the pre-existing, both private and public, hospitals (and other health structures) network.

Our study is focused on the reorganization of the public hospitals network in Calabria. The regional network is based on a “Hub and Spoke” topology.

The proposed formulation is aimed at identifying the hospitals (nodes) that will constitute the spokes of the network, by taking into account the regional reorganization guidelines. The regional authority aims to rationalize the health network, by adjust the regional capacity (in terms of number of beds) to appropriate values, which meet the national reference values.

The reconfiguration process is realized also through a reduction of ordinary admissions, by transforming part of them to day hospitals and/or domestic assistance and by transforming parts of the actual hospitals to Territorial Primary Assistance Centers (Centri di Assistenza Primari), supplying fundamental care services. This process leads to a reduction of the number of hospitals to a minimum value, satisfying appropriate assistance services. To this end, the scope of the proposed model is to minimize the number of spokes to be activated, while guaranteeing an appropriate level of service (measured in terms of satisfied demand) and considering the national and local authorities requirements.

On the basis of the previous considerations, the model takes into account the following issues:

- the number of beds available on the reference territory. In our cases the national rules define a “target” number of 4 beds per 1000 inhabitants;
- a minimum total number of beds available in each of the departments and determined by considering all the chosen spokes; this requirement is necessary to satisfy the territorial demand for specific services;
- a given number of beds present in each spoke; in our case we take into account the values associated with each spoke by regional authorities, determined in such a way to be coherent with national standards.

The model can be also enriched with additional constraints in order: to guarantee that a minimum number of beds (capacity) is available in each of the hospitals; to ensure that in all

the hospitals specific departments are present, since hospital to be classified as spoke needs to satisfy precise requirements.

To describe the model for the public hospitals network reorganization problem, we need to introduce the following additional notations:

- Parameters

- PL^k : the number of beds of department k , $k = 1, \dots, r$ to be activated in the considered area;
- c_j : the available capacity (i.e., number of beds) of hospital j , $j = 1, \dots, n$;
- c_T : the total capacity, i.e. the number of beds to be opened in the considered area;
- M : a big positive number;
- δ_{kj} : equal to 1 if departments k is already present in hospital j , 0 otherwise.

- Variables

- x_{kj} : a non-negative integer variable equal to the number of beds assigned to the department k of hospital j ;
- s_{kj} : binary variable telling if department k is opened in hospital j . In particular $s_{kj} = 1$ if $x_{kj} = 1$, $s_{kj} = 0$ otherwise.

The mathematical formulation of the hospital network reorganization problem (HNRP_a) can be stated as follows.

$$\min \quad \sum_{j=1}^n y_j \quad (26)$$

$$\sum_{k=1}^r x_{kj} \leq c_j \quad \forall j = 1, \dots, n \quad (27)$$

$$\sum_{j=1}^n x_{kj} \geq PL_k \quad \forall k = 1, \dots, r \quad (28)$$

$$\sum_{k=1}^r \sum_{j=1}^n x_{kj} \leq c_T \quad (29)$$

$$y_j \leq \sum_{k=1}^r x_{kj} \quad \forall j = 1, \dots, n \quad (30)$$

$$\sum_{k=1}^r x_{kj} \leq M y_j \quad \forall j = 1, \dots, n \quad (31)$$

$$x_{kj} \geq 0, \quad \forall k = 1, \dots, r \quad j = 1, \dots, n \quad (32)$$

$$y_j \in (0, 1), \quad \forall j = 1, \dots, n \quad (33)$$

The objective function (26) is to minimize the number of hospitals to be opened. The constraints (27) - (29) represent all the requirements, imposed by national and regional rules.

Conditions (27) represent hospital capacity constraints. Constraints (28) refer to the necessity of having, for each department, a minimum number of beds available in all the hospitals that will be opened in the area considered. Constraints (29) impose that the total number of beds to be activated in the area considered do not exceed the settled value c_T (i.e., 4 beds per 1000 inhabitants).

Constraints (30) and (31) state that a department can be opened only in an activated hospital. Constraints (32) and (33) impose, respectively, that x are non-negatives and y are binary variables.

Variants of the model (26)-(33) can be derived by introducing the following optional groups of constraints:

$$x_{kj} \geq \alpha s_{kj} \quad \forall k = 1, \dots, r; \quad j = 1, \dots, n \quad (34)$$

$$s_{kj} \leq x_{kj} \quad \forall k = 1, \dots, r; \quad j = 1, \dots, n \quad (35)$$

$$x_{kj} \leq M s_{kj} \quad \forall k = 1, \dots, r; \quad j = 1, \dots, n \quad (36)$$

$$\sum_{k=1}^r s_{kj} \geq \beta y_j \quad \forall j = 1, \dots, n \quad (37)$$

$$x_{\bar{k}j} \geq y_j, \quad \forall j = 1, \dots, n \quad (38)$$

$$x_{kj} \leq M \delta_{kj} + \gamma(1 - \delta_{kj}) \quad \forall k = 1, \dots, r; \quad j = 1, \dots, n \quad (39)$$

with α, β e γ positive integers.

Constraints (35) and (36) are logical links between variables s and x and together with constraints (34) set a minimum numbers of beds α , to be activated in each department; constraints (37) impose that if hospital is opened it must contain a minimum numbers of departments equal to β ; constraints (38) impose that department $\bar{k}$ is opened in each active hospital; constraints (39) ensure that if a department k is not currently present in hospital j , it can be opened in j , but a restriction on the maximum number of beds equal to γ is imposed.

The (HNRP_a) model has been defined to take into account the national guidelines and constraints, without any explicit consideration of the demand.

In what follows, a model (HNRP_d) aimed at maximizing the satisfied demand, if some of the hospitals of the existing network are closed, is presented. In particular, it is assumed that only p hospitals must remain open and the objective is to reallocate parts of the initial demand on the new health care network. It is also assumed that the capacity of the hospital departments (which will still be active after the reorganization) can be increased by a certain quantity, while the overall capacity of each hospital remains unchanged.

In order to describe the proposed model, it is useful to introduce the following additional notations:

- Parameters

- l_j^k : the number of beds of department k in hospital j ;

- α_j^k : the percentage of extra beds admitted in department k of hospital j ;
- d_j^k : the demand of department k at hospital j ;
- $a_{jj'}$ equal to 1 if the distance between j and j' is less than S ; 0 otherwise.

• Variables

- $x_{jj'}^k$: the fraction of demand related to department k in hospital j to be moved in hospital j' ;
- e_j^k : the extra capacity (extra number of beds) admitted in department k of hospital j ;

The problem (HNRP_d) can be mathematically represented as follows:

$$\max_x \quad \sum_{j=1}^n \sum_{j'=1}^n \sum_{k=1}^r a_{jj'} d_j^k x_{jj'}^k \quad (40)$$

$$\sum_{j=1}^n \sum_{k=1}^r d_j^k x_{jj'}^k \leq c_{j'} y_{j'} \quad \forall j' = 1, \dots, n \quad (41)$$

$$\sum_{j=1}^n d_j^k x_{jj'}^k \leq e_{j'}^k + l_{j'}^k \quad \forall j' = 1, \dots, n, \quad k = 1, \dots, r \quad (42)$$

$$e_{j'}^k \leq \alpha_j^k l_{j'}^k \quad \forall j' = 1, \dots, n, \quad k = 1, \dots, r \quad (43)$$

$$\sum_{j=1}^n y_j \leq p \quad (44)$$

$$x_{jj'}^k \leq l_{j'}^k \quad \forall j = 1, \dots, n, \quad j' = 1, \dots, n, \quad k = 1, \dots, r \quad (45)$$

$$x_{jj'}^k \leq y_j \quad \forall j = j', j = 1, \dots, n, \quad j' = 1, \dots, n, \quad k = 1, \dots, r \quad (46)$$

$$x_{jj'}^k \leq 1 - y_j \quad \forall j \neq j', j = 1, \dots, n, \quad j' = 1, \dots, n, \quad k = 1, \dots, r \quad (47)$$

$$\sum_{j'=1}^n x_{jj'}^k \leq 1 \quad \forall j = 1, \dots, n, \quad k = 1, \dots, r \quad (48)$$

$$x_{jj'}^k \leq a_{jj'} \quad \forall j = 1, \dots, n, \quad j' = 1, \dots, n, \quad k = 1, \dots, r \quad (49)$$

$$x_{jj'}^k \geq 0, \quad \forall k = 1, \dots, r, \quad j = 1, \dots, n, \quad j' = 1, \dots, n \quad (50)$$

$$e_j^k \geq 0, \quad \forall k = 1, \dots, r, \quad j = 1, \dots, n \quad (51)$$

$$y_j \in (0, 1), \quad \forall j = 1, \dots, n \quad (52)$$

The objective function (40) maximizes the covered (within S) demand, when a maximum number p of hospital remains opened. Equations (41) are capacity constraints on each hospitals, while constraints (42) and (43) allow the possibility of increasing the capacity of the department,

in the activated hospitals, of a certain percentage α_j^k of the actual capacity. Constraints (44) limit the number of opened hospitals to p . Equations (45) are capacity constraints on each departments of the opened hospitals. Constraints (46) and (48) impose that the demand satisfied by a given hospital, before the reorganization, has to be assigned to the same hospital, if it is still open. Constraints (48) allow the possibility of a partial satisfaction of the demand, while constraints (49) are of the coverage type.

4 Computational Experiments

In this section the computational results obtained by testing the models described in Sections 2 and 3 are presented.

4.1 The facility location problems

The optimization models described in Section 2 have been tested on real data provided by the local healthcare authority “Azienda Sanitaria Provinciale (ASP)” of Cosenza, a city in the region of Calabria, located in the south of Italy (the green region in Figure 1). Figure 2 shows the hospitals location in the case of our interest.

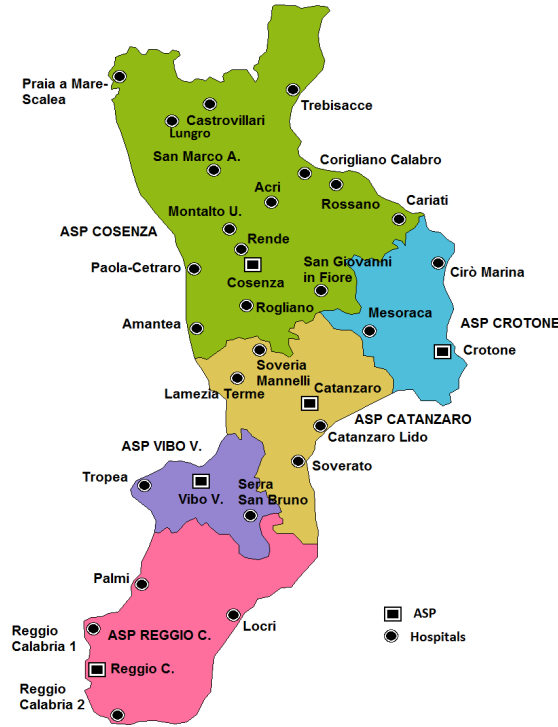


Figure 1: Healthcare configuration of the Calabria region

The ASP, together with the “Azienda Ospedaliera di Cosenza (AO)”, coordinate the public provincial healthcare activities, related to the management of public hospitals and other qualified structures, such as care homes, clinical laboratories, home for the disabled and the aged.

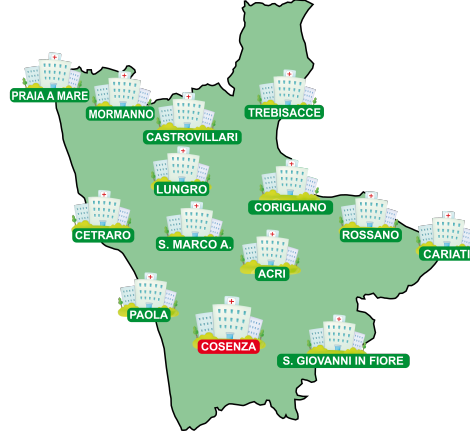


Figure 2: Healthcare network of the Province of Cosenza

The Cosenza ASP includes thirteen hospitals, as reported in Table 1, in which the name of the city, where each hospital is located, is given. The list of the departments belonging to the hospitals is reported in Table 2. The AO is formed by the Annunziata and the Mariano Santo hospitals, localized in Cosenza, and the Santa Barbara hospital in Rogliano.

Hospitals	
1	Acri
2	Castrovillari
3	Cetraro
4	Corigliano
5	Lungro
6	Mormanno
7	Paola
8	Praia a mare
9	Rossano
10	Trebisacce
11	San Giovanni in Fiore
12	San Marco Argentano
13	Cariati

Table 1: Hospitals belonging to the Cosenza ASP

The ASP guarantees primary assistance over all the territory, by the sanitary districts. At present, six sanitary districts are active (155 municipalities), as reported in Table 3.

A district is identified on the basis of a population of minimum 60,000 inhabitants, unless particular geographical and morphological conditions or area with a lower population density. Districts are economically and managerially autonomous and ensure, to the reference population, access to the primary outpatient assistance and to some social services such as female, addiction and older support.

In the scenario considered, we assume that the demand centers coincide with the given sanitary districts. Consequently, for each demand center i and for each department k , d_i^k represents the demand coming from the i -th sanitary district operating in the Cosenza province. Thus, d_i^k is obtained by aggregating the number of patients admitted to the k -th department of all the considered hospitals and coming from a city belonging to district i . In order to evaluate the travel time between the demand center and the hospital location, each district is identified with the most populous city belonging to it, as shown in Table 4.

For each hospital j and for each department k , the parameter l_j^k , representing the capacity of the department k of the hospital j , was determined by considering the average hospital length

Departments	
1	Cardiology
2	General surgery
3	Geriatric ward
4	General medicine
5	Nephrology
6	Nursery
7	Neurology
8	Ophthalmology
9	Orthopedics
10	Obstetrics and Gynecology
11	Otorhinolaryngology
12	Pediatrics and neonatology
13	Psychiatry
14	Urology
15	Intensive Care
16	Cardiological Intensive Care
17	Intensive Rehabilitation
18	Long-Term Care
19	Oncology
20	Pneumology
21	Multidisciplinary
22	Rheumatology

Table 2: Departments

Sanitary District	
1	Cosenza Savuto
2	Jonio Nord
3	Jonio Sud
4	Pollino Esaro
5	Tirreno
6	Valle Crati

Table 3: Sanitary districts belonging to the Cosenza ASP

of stay (AHLoS)(i.e. the average number of days a patient spends in the k -th department) and applying the following formula $l_j^k = \frac{\bar{l}_j^k \times T_Y}{AHLoS}$ where $\bar{l}_j^k$ is the number of available beds in the department k of hospital j and T_Y , the number of days per year is set to 365. It is worth noting that, in the following numerical experiments, the number of active departments is set to 20, since in the years of interest, departments 21 and 22 have a demand equal to zero.

Demand centres	
1	Cosenza from Valle Crati District
2	Rende from Jonio Nord District
3	Rossano from Jonio Sud District
4	Corigliano from Jonio Nord District
5	Castrovillari from Pollino Esaro District
6	Paola from Tirreno District

Table 4: Districts and Centres

We established, by solving the model Covering_HLP1, that the number of hospitals necessary to satisfy the demand with the current capacity is 9. By imposing $p = 9$ in the pMedian_HLP, pCenter_HLP, and Covering_HLP2 models, we obtain the values of the objective function reported in Table 5, where the travel times are expressed in minutes, whereas the distances are given in kilometres. For the model Covering_HLP2, we have considered two different values of the coverage-distance S , i.e. $S = 50$ and $S = 100$ kilometres. Furthermore, since the demand could not be completely satisfied, column S_d of Table 5 reports the percentage of the demand satisfied. From Table 5, it is evident that this value increases as the coverage-distance increases and for $S = 100$ km it is close to 1.

The results of Table 6, where the value of the variables y is reported, underlines that almost all models suggest that hospitals 1, 2, 3, 4, 5, 6, 9, 10, 13 should be opened, with the exception of Covering_HLP2, with $S = 50$, which activates hospitals 7 and 12 instead of hospitals 5 and 6.

Models	Objective function	S_d
pMedian_HLP	4096.677	
pCenter_HLP	45367.159	
Covering_HLP1	9	
Covering_HLP2 S=50	27179	92.30%
Covering_HLP2 S=100	29422	99.90%

Table 5: Objective function and S_d values

y_j	1	2	3	4	5	6	7	8	9	10	11	12	13
pMedian_HLP	1	1	1	1	1	1			1	1			1
pCenter_HLP	1	1	1	1	1	1			1	1			1
Covering_HLP1	1	1	1	1	1	1			1	1			1
Covering_HLP2 S=50	1	1	1	1			1		1	1		1	1
Covering_HLP2 S=100	1	1	1	1	1	1			1	1			1

Table 6: Variables y_j values

To assess the efficiency and effectiveness of the solutions obtained, the following indicators have been computed:

- index $R(j)$, indicating the capacity factor of the departments in hospital j . It is the number of activated departments on the overall available departments of hospital j . A value of $R(j)$ equal to 1 indicates that all the available departments are opened in hospital j ;
- index $PL(j, k) = \frac{\sum_{i=1}^m d_{ik} x_{ij}^k}{l_j^k} \times 100$, representing the utilization of beds in department k of hospital j . It is the ratio between the demand satisfied by department k of hospital j and the number of beds of department k in hospital j .

Table 7 reports the value $R(j)$ for each hospital j . From the collected results, it is evident that the values of $R(j)$ are less than 0.5 only in very few cases, indicating that most of the departments are necessary to meet the demand.

$R(j)$	1	2	3	4	5	6	7	8	9	10	11	12	13
pMedian_HLP	1	1	1	1	0.75	1			1	0.67			0.33
pCenter_HLP	1	1	0.80	1	1	1			0.80	0.83			0.50
Covering_HLP1	1	1	0.8	0.90	0.75	1			0.60	0.33			0.50
Covering_HLP2 S=50	1	1	1	1			1		0.60	0.67		0.80	0.67
Covering_HLP2 S=100	1	1	0.90	0.90	0.75	1			0.50	0.67			0.33

Table 7: Index $R(j)$ values

From Tables 8-12, containing the value of $PL(j, k)$, information, can also be obtained, on the utilization of department k in hospital j . To take an example of the possible information that can be obtained from the indexes $R(j)$ and $PL(j, k)$, we can observe that hospital 13, having the lowest values of $R(j)$, also presents a limited number of activated departments and/or almost the departments are underused. This observation suggests that hospital 13 could be closed, while redistributing the demand conveniently.

Tables 8 -12 underline that the values of $PL(j, k)$ are high in the majority of cases. The most underused departments are 8, 11 and 19. The utilization of the capacity in these departments is very low, indicating that the decision maker could consider the possibility of closing them. For Covering_HLP2 model, we also report the value of the $S_d(k)$, the percentage of the demand satisfied by departments k . From Table 12, it is evident that the higher the coverage-distance, the higher the value of $S_d(k)$.

To gain some insights on the post reorganization scenario, the Covering_HLP2 models with S=50, 100 and 500, were solved by imposing that the variables y_j are equal to 1 for all the

spokes at present open and reported in Table 13. The aim is to have some information on the demand that can be satisfied with the current health services network. By comparing Tables 5 and 14, can be observed that in the post-reorganization configuration, a reduction of the satisfied demand S_d is observed even if the coverage constraints ($S = 500$) are redundant. On the other hand, the values of $R(j)$ in Table 15 highlight that, since a lower number of hospitals is activated, the post-reorganization scenario requires higher number of departments in each of the open hospitals.

By comparing Tables 11 and 12 with Tables 16, 17 and 18 and remembering that hospitals 5, 6 and 7 in Table 13 are indicated with indexes 7, 9 and 11 in Table 4, the following considerations can be made:

- in the hospital 1 (i.e., Aciri) only one department operates after the reorganization. This specific situation can be explained by observing that Aciri is a mountain-hospital where only department 4, General Medicine, is activated after reorganization;
- department 3 is not opened after the reorganization, because it is not compulsory in a spoke and its activity (old age people treatment) can be transferred to other structures, without too much effort;
- department 6, nursery, does not present a good percentage of satisfied demand, because it was closed in the major of hospitals after reorganization;
- departments 11 and 17, otolaryngologist medicine and intensive rehabilitation have not been assigned beds after the reorganization;
- all the remaining departments present a good percentage of satisfied demand.

The considerations we made are only an example of how the facility location models and the defined indicator could be used by decision makers, if properly integrated in a Decision Support System. It is important to underline that with the current network configuration some inefficiencies appear. Indeed by applying the facility location models of Section 2, we established that the minimum number of hospitals needed to satisfy the demands is $p = 9$. Nevertheless by imposing the current network configuration, made up of 7 hospitals, we obtain the percentage of satisfied demand reported in Table 14. Although, at first sight, the percentage of satisfied demand seems quite good, it is important to observe that, in the sector of our interest, having some of the demand uncovered (9%) is an unbearable social cost.

4.2 The health care network reorganization problems

In this section we test the models defined in Section 3 on a subset, but sufficiently meaningful, of data relative to public hospitals (the public capacity in terms of number of beds is about the 70% of the overall capacity). We will concentrate our attention on one of the three area affected by the reorganization process, namely the northern Area made up of the province (territory) of Cosenza. The hospitality network is Hub and Spoke where the “Hub”, as identified by the regional authorities, is the “Azienda Ospedaliera di Cosenza (AO)”. This choice is due to the appropriate dimensions (capacity), type and variety of supplied services and location of (AO). We will also assume that the AO is the hub node of the network.

4.2.1 The HNRP_a formulations

To test the HNRP_a model and its variants, we first considered the following two cases:

- Case HNRP_a1: Model HNRP without any restriction on the hospitals to be activated.
- Case HNRP_a2: Model HNRP with the constraint that hospitals 1 and 11 (mountain area hospitals) must be open.

Tables 19-21 show the values of the variables y_j and x_{jk} variables for HNRP_a1 and HNRP_a2. Considering that the solutions obtained are quite similar and that the constraint on the presence of the two mountain hospitals is hard, in the sequel we will concentrate our attention only on the HNRP_a2 case.

Furthermore, we consider the complete version of the HNRP_a model defined by the objective function (26) and constraints (27)-(39). In particular, two versions of HNRP_a2 are considered. In both the versions we set α , i.e. the minimum number of beds to be activated in each departments, equal to 4; the minimum numbers of departments to activate in each hospital, β , equal to 4; the departments $\bar{k}$ equal to 2 and 4 opened in each activated hospital (this two departments are compulsory in spoke nodes). The two versions differ in the setting of γ representing the maximum number of beds that must be opened in a department that is new in an activated hospital j . In particular in HNRP_a21 $\gamma = 9$ whereas in HNRP_a22 $\gamma = 13$.

Table 22 reports the value of the objective function and the variables y_j for HNRP_a21 and HNRP_a22 together with the values of α , β , γ and $\bar{k} = 2, 4$. Tables 23 and 24 show the values of the variables x_{jk} in the HNRP_a21 and HNRP_a22 cases.

The number of beds, corresponding to the solutions of HNRP_a2, HNRP_a21 and HNRP_a22, are then used as input for solving the model Covering_HLP2 described in Section 2. The aim is to determine the percentage of the demand satisfied given the current (post-reorganization) number of beds. The results obtained, in terms of $S_d(k)$, y_j , $R(j)$ and $PL(j, k)$, are reported in Tables 25 –32.

It is worth noting that in Table 25 and in the case of $S = 50$ some of the hospitals are not opened due to the fact that they are not within the coverage-distance of 50 km. Moreover, the percentage of satisfied demand is quite modest due to the fact that HNRP_a2, HNRP_a21 and HNRP_a22 models do not take into account the demand.

From Table 26, it is clear that all the departments are used in the open hospitals. With reference to $PL(j, k)$, representing the use of beds in department k of hospital j , reported in Tables from 27 to 32, two different cases deserve the attention of the decision maker:

1. high values of $PL(j, k)$ associated with low value of $S_d(k)$ that suggests that the department k is under dimensioned;
2. low values of $PL(j, k)$ associated with high value of $S_d(k)$ that suggest that departments k could be closed in some of the active hospitals.

It is important to underline that models HNRP_a2, HNRP_a21 and HNRP_a22 were constructed by considering the reorganization guidelines of national and regional authorities and, in fact, they produce a network configuration similar to that currently adopted. Indeed the solution adopted from the regional authorities foresee the activation of the following spokes: Castrovillari (2); Corigliano–Rossano(4–9); Paola–Cetraro (7–3); Mountain Acri (1); Mountain S.Giovanni in Fiore (11); Rehabilitative hospital Mormanno (6).

The solution obtained from HNRP_a2 with $S = 100$ activate the following spokes Mountain Acri (1); Mountain S.Giovanni in Fiore (11); Castrovillari (2), Cetraro (3), Corigliano (4); Paola (7), Rossano(9).

The solution obtained from HNRP_a21 with $S = 100$ suggests to open all the hospitals except Lungro (5), Mormanno (6) and Trebisacce (10).

The solution obtained from HNRP_a22 with $S = 100$ is: Mountain Acri (1); Mountain S.Giovanni in Fiore (11); Castrovillari (2); Corigliano (4); Paola (7), Rossano(9) and Trebisacce (10).

It is evident that, in all the cases, the unsatisfied demand (a percentage of 20% in the best of the cases) is extremely high if we take into account the type of service under consideration. This situation is probably due to the fact that the demand was not properly and explicitly considered in defining the post-reorganization network.

4.2.2 The HNRP_d formulation

In order to test the HNRP_d model, we have considered the demand and the number of beds available before the reorganization process. We defined different instances by varying the parameters p (set to $9 - 7 - 5 - 3$), α_j^k (set to $0 - 0.25 - 0.50$) and S (set to $50 - 100$).

The results obtained are given in Table 33, where we report the active hospitals (y_j) and the percentage of satisfied demand (S_d) for each of the scenarios under consideration.

We can notice that with $p = 9$ hospitals, all the demand will be satisfied, without the need to add more capacity in any of the departments ($\alpha = 0$). With respect to the current regional network configuration, our solution does not foresee the activation of 11 but activates 5 and 8. It is worth noting that reducing p will cause a reduction of the satisfied demand. Furthermore, a certain increase can be also obtained by incrementing α , while keeping p fixed. By the application of our model we obtain, in any case, a complete satisfaction of the demand by properly identifying the hospitals that must be open and by slightly adjusting the available capacity.

Further adjustment could be made by the decision makers by analysing $PL(j, k)$. Tables 34-37 show the values of index $PL(j, k)$ with $p = 9, 7, 5, 3$, $S = 100$ Km and $\alpha_j^k = 0$. It can be seen that there are some departments in which the degree of utilization of beds is very low indicating that the possibility of closing this department could be considered. For example, for department 8 this index shows a value less than 1 in almost all the hospitals, indicating that the demand for this department is much lower than the available capacity. In this case it could be more appropriate to close department 8 while managing properly the new generated unsatisfied demand.

In order to better understand the influence of the parameter α_j^k , in Tables 38 and 39 we have reported the values of $PL(j', k)$ for $p = 7$, $S = 100$ Km and both $\alpha_j^k = 0.25$ and $\alpha_j^k = 0.50$.

By comparing these values with the case $\alpha_j^k = 0$, given in Table 35, it is evident that when $\alpha_j^k = 0.50$, the values of $PL(j, k)$ are lower because the extra capacity α_j^k is set to a value that is too high and, in fact, a good value for α_j^k seems to be 0.25. Indeed, for example, the use of beds in department 16 with $\alpha_j^k = 0$ is high in hospitals 2 and 9, while the degree of utilization for beds in hospital 3 is less than 50%. An extra capacity for hospital 2, will cause the closure of department 16 in hospital 3, and an increasing of the percentage of utilization of beds in department 2 from 50.62% to 99.75%, and a small decreasing of the percentage in hospital 9 from 97.65% to 96.00%. The above considerations are just an example of possible use of our

models to construct and discuss different alternative solutions by properly varying some of the parameters.

It is important to underline that the definition and the application of the proposed HNRP_d model can lead to an improvement in the supply of healthcare services, by varying the structure of the network properly by closing some of the hospitals and by conveniently adjusting the departments capacity, complying with the total network capacity.

5 Conclusions

In this paper, we described the reorganization problems faced by the local authorities in defining the hospitals network of the northern area of region Calabria.

On the basis of real data, we compared the current hospital network with the ideal localization obtained by solving some classical facility location models. The results show that the current hospital network configuration leaves some of the demand uncovered thus generating inefficiencies for the healthcare system.

We also considered the problem of reorganizing the public health care service network by proposing two ad-hoc optimization models to support the decision-making process: a model that, taking into account national and regional guidelines and constraints, minimizes the number of open hospitals; and a model that allow the possibility of reconfiguring the current network by closing some of the hospitals and by partially redistributing the available capacity. We applied the two models to real data: the first model, not explicitly taking into account the demand, generates a network configuration with an unacceptable percentage of unsatisfied demand. The second model guarantees the complete satisfaction of the demand and the possibilities, for decision makers, of studying different alternative solutions obtained by properly varying the parameters.

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$PL(j, k)$		pMedian_HLP p=9																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		27.65	96.39	100.00		100.00			16.86			27.31								
2	93.48	100.00	100.00	91.77		91.77		1.70	100.00	55.47		89.29	100.00		100.00	100.00			0.51	53.12
3	12.09	72.63	100.00	100.00		100.00		0.36	21.31	21.31		88.74	67.23	51.90	90.28	98.91				
4		100.00	100.00	100.00		100.00	25.57		83.18	93.99	7.85	84.26	100.00	87.86						
5			44.87	100.00													100.00			
6			100.00	100.00													76.46	96.48		
7																				
8																				
9	71.63	82.50		71.39	71.17	100.00		0.36		64.85		100.00				64.08			1.62	
10		70.10		35.02		100.00										89.80				
11																				
12																				
13			3.85			100.00														

Table 8: $PL(j, k)$ under the pMedian_HLP model

$PL(j, k)$		pCenter_HLP p=9																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		100.00		100.00		100.00			17.31			100.00								
2	100.00	100.00		100.00		91.77		2.50	75.54	68.96		100.00	68.91		100.00	100.00			2.53	23.56
3	18.37	46.69		100.00		100.00						100.00	67.23	100.00	90.28	100.00				
4		96.46		91.42		100.00	25.37		100.00	87.64	7.85	75.38	42.86	15.76						
5			46.79	100.00														100.00		31.14
6				100.00													76.46	96.48		
7																				
8																				
9		49.90		71.39	71.17	100.00		0.24		69.82		82.96				67.76				
10	19.33	40.95		38.61		100.00										84.49				
11																				
12																				
13	49.73	28.89				100.00														

Table 9: $PL(j, k)$ under the pCenter_HLP model

$PL(j, k)$		Covering_HLP1																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	100.00	100.00		100.00		100.00				100.00		100.00								
2	100.00	100.00		100.00		100.00		2.90	100.00	100.00		100.00	100.00		100.00	100.00			2.53	53.12
3	73.95	100.00		100.00						95.23		100.00		100.00	90.28	100.00				
4	100.00	100.00		100.00		100.00	25.57		83.18		7.85	64.90	94.54	15.76						
5				100.00														97.41		
6				100.00													76.46	100.00		
7																				
8																				
9		29.10		52.53	71.17	83.52						100.00				52.24				
10						100.00										100.00				
11																				
12																				
13		100.00		90.61		100.00														

Table 10: $PL(j, k)$ under the Covering_HLP1 model

$PL(j, k)$		Covering_HLP2 p=9 S=50																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	100.00	100.00		100.00		100.00				100.00			100.00							
2	56.37	100.00		100.00		98.72		1.10	76.57	58.08		97.86	85.71		100.00	100.00		2.02		4.83
3	100.00	100.00		100.00				0.66		21.31		88.74	67.23	50.69	56.48	98.91				
4	100.00	100.00		100.00		58.61	23.91		37.50	100.00	6.99	100.00	34.45	78.55						
5																				
6																				
7	11.16	16.38	17.90	11.31					89.88									0.51		
8																				
9	28.14				65.24	100.00		0.24				30.26				37.55				
10		36.65		58.13		100.00										100.00				
11																				
12		100.00		100.00				0.90				100.00								
13		36.10	3.85	100.00		100.00														
$S_d(k)$	100.00%	100.00%	100.00%	100.00%	92.00%	97.00%	93.00%	100.00%	100.00%	100.00%	89.00%	99.00%	100.00%	93.00%	74.00%	96.00%		100.00%	100.00%	9.00%

Table 11: $PL(j, k)$ under the Covering_HLP2 model and S=50

$PL(j, k)$		Covering_HLP2 p=9 S=100																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	100.00	100.00		18.13		100.00				100.00			76.89							
2	82.61	100.00		97.63		100.00		2.90	97.66	96.90		100.00	85.71		74.07	100.00		2.53		53.12
3	100.00	70.56		100.00						100.00		54.36	100.00	100.00	100.00	100.00				
4	100.00	100.00		100.00		100.00	25.57		84.79		7.85	100.00	24.79	15.76						
5			46.79															97.41		
6				66.76													74.07	100.00		
7																				
8																				
9				93.74	66.89	83.52						100.00				100.00				
10		58.43		100.00		100.00										52.24				
11																				
12																				
13	100.00					100.00														
$S_d(k)$	100.00%	100.00%	100.00%	100.00%	94.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	97.00%	100.00%	100.00%	100.00%

Table 12: $PL(j, k)$ under the Covering_HLP2 model and S=100

Hospitals	
1	Acri
2	Castrovillari
3	Cetraro
4	Corigliano
5	Paola
6	Rossano
7	San Giovanni In Fiore

Table 13: Hospitals in the current network configuration (post-reorganization)

Covering_HLP2	Objective function	S_d
S=50	23217	78.80%
S=100	26795	91.00%
S=500	26821	91.10%

Table 14: Objective function and S_d values under the current network configuration

R_j	Covering_HLP2	1	2	3	4	5	6	7
S=50	1	1	0.88	1	1	0.88		
S=100	1	0.92	0.88	0.83	0.67	0.88	1	
S=500	1	1	0.88	0.83	0.67	0.88	1	

Table 15: R_j under the current network configuration

$PL(j, k)$		Covering_HLP2 S=50																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1				100.00																
2	61.42	100.00		100.00		100.00		1.65	100.00	100.00		100.00	33.75		100.00	100.00			0.61	9.67
3							3.12			13.05		81.37	43.40	60.87	100.00		100.00			
4		100.00		100.00			21.83			72.61		43.62	33.96							
5	74.22	86.73		100.00					65.91							82.41			0.12	
6	28.14				78.26				58.01					23.60	75.93	42.78		20.37		
7																				
$S_d(k)$	100.00%	83.00%		84.00%	92.00%	24.00%	98.00%	38.00%	99.00%	94.00%		94.00%	91.00%	64.00%	88.00%	83.00%		41.00%	100.00%	9.00%

Table 16: $PL(j, k)$ under Covering_HLP2 model, S=50 and the current network configuration

$PL(j, k)$		Covering_HLP2 S=100																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1				100.00																
2	100.00	100.00		100.00		100.00		4.35		13.85		100.00	100.00		100.00	62.09			0.76	100.00
3							25.57			65.40		37.08		62.32	100.00		100.00			
4		100.00		100.00						86.71		100.00	22.01			100.00				
5		97.93		100.00					100.00											
6	86.28				80.24				62.06					70.39	100.00	100.00		100.00		
7				96.53																
$S_d(k)$	100.00%	100.00%		100.00%	94.00%	24.00%	100.00%	100.00%	100.00%	100.00%		100.00%	100.00%	100.00%	98.00%	100.00%		69.00%	100.00%	94.00%

Table 17: $PL(j, k)$ under Covering_HLP2 model, S=100 and the current network configuration

$PL(j, k)$		Covering_HLP2 S=500																		
j \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1				100.00																
2	100.00	100.00		96.99		100.00		4.35	100.00	13.85		100.00	100.00		100.00	100.00			0.76	100.00
3							25.57			65.40		37.08		100.00	100.00		100.00			
4		97.93		100.00						86.71		100.00	22.01							
5		100.00		100.00					100.00							76.28				
6	86.28				85.38				25.36					32.71	100.00	100.00		100.00		
7				100.00																
$S_d(k)$	100.00%	100.00%		100.00%	100.00%	24.00%	100.00%	100.00%	100.00%	100.00%		100.00%	100.00%	100.00%	98.00%	100.00%		69.00%	100.00%	94.00%

Table 18: $PL(j, k)$ under Covering_HLP2 model, S=500 and the current network configuration

Hospital locations	1	2	3	4	5	6	7	8	9	10	11	12	13
HNRP_a1	y_j	1	1	1	1				1	1			
HNRP_a2	y_j	1	1	1	1				1		1		

Table 19: y_j values

x_{jk}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1														20	10	19	30			6		
2										61		52								4		48
3									71	19									19			
4				30	10		20												9		87	
5																						
6																						
7				93														26				
8																						
9	8	91																44				
10	23																					
11																						
12																						
13																						

Table 20: x values for the HNRP_a1 model

x_{jk}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1														20	10	19	30			6		
2										61		52								4		48
3									71	19									19			
4				30	10		20												9		87	
5																						
6																						
7				93														26				
8																						
9	8	91																44				
10																						
11	23																					
12																						
13																						

Table 21: x values for the HNRP_a2 model

Models	Objective function	y_j	α	β	γ	k
HNRP_a21	10	1,2,3,4,7,8,9,11,12,13	4	4	9	2,4
HNRP_a22	7	1,2,4,7,9,10,11	4	4	13	2,4

Table 22: Objective functions, y_j and parameters values for HNRP_a21 and HNRP_a22 models

x_{jk}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		4		36			9										9	9			9	9
2		4		17					10	80				9	10		8	9			9	9
3		55		35													9				9	
4		4		4					61			48		4			4	9	4		9	9
5																						
6																						
7		4		4			4							7				9	24	6	9	8
8		4		4													9				9	
9	4	4		4	10		7					4				19	9	4		4	9	4
10																						
11		4		4														4			9	9
12		4		4														8			6	
13	27	4		11																	9	

Table 23: x values for HNRP_a21 model

x_{jk}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		4		46													13	13			9	
2		4		26					12	80		4						13			13	13
3																						
4		4		35								48		7			13	13		10	13	13
5																						
6																						
7		4		4	10		7		59									13			13	9
8																						
9	31	7		4			13							13	10	15	4	5	28		13	
10		36		4												4					13	
11		32		4														13			13	13
12																						
13																						

Table 24: x values for HNRP_a22 model

Models	Objective function	y_j	S_d
Covering_HLP2_HNRP_a2 S=50	12069	1,2,3,4,7,9	41.00%
Covering_HLP2_HNRP_a2 S=100	24208	1,2,3,4,7,9,11	82.20%
Covering_HLP2_HNRP_a21 S=50	16694	1,2,3,4,7,9,12,13	56.70%
Covering_HLP2_HNRP_a21 S=100	23682	1,2,3,4,7,8,9,11,12,13	80.40%
Covering_HLP2_HNRP_a22 S=50	16493	1,2,4,7,9,10	56.00%
Covering_HLP2_HNRP_a22 S=100	23716	1,2,4,7,9,11,10	80.50%

Table 25: Objective function values, activated hospitals and satisfied demand percentage under Covering_HLP2 model

R_j	1	2	3	4	5	6	7	8	9	10	11	12	13
Covering_HLP2_HNRP_a2 S=50	1	1	1	1			1		1				
Covering_HLP2_HNRP_a2 S=100	1	1	0.667	0.750			1		1		1		
Covering_HLP2_HNRP_a21 S=50	1	0.875	1	1			1		0.600			1	1
Covering_HLP2_HNRP_a21 S=100	1	0.875	1	0.875			0.625	1	0.700		0.500	1	1
Covering_HLP2_HNRP_a22 S=50	1	0.857		1			1		1	1			
Covering_HLP2_HNRP_a22 S=100	1	0.857		1			0.857		0.857	1	0.750		

Table 26: $R(j)$ under Covering_HLP2 model

$PL(j, k)$		Covering_HLP2_HNRP_a2 S=50																					
$j \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1														19.96	50.00	31.24				100.00			
2										44.05		24.46									100.00		
3									24.04	20.60									0.21	24.12		4.76	
4				100.00	80.24		11.97												1.80				
5																							
6																							
7				58.44														100.00					
8																							
9	71.63	42.98																9.25					
10																							
11																							
12																							
13																							
$S_d(k)$	32.00%	35.00%		55.00%	94.00%		93.00%		35.00%	64.00%		32.00%		30.00%	49.00%	35.00%		52.00%	100.00%	66.00%		9.00%	

Table 27: $PL(j, k)$ under Covering_HLP2_HNRP_a2 model and S=50

$PL(j, k)$		Covering_HLP2_HNRP_a2 S=100																					
$j \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1																							
2										79.34		75.82		66.29	100.00	90.02				100.00			
3									68.47														
4				100.00	85.38		12.80												1.07				
5																							
6																							
7				100.00														79.54					
8																							
9	40.93	100.00																85.07					
10																							
11	63.64																						
12																							
13																							
$S_d(k)$	100.00%	83.00%		80.00%	100.00%		100.00%		100.00%	100.00%		100.00%		100.00%	98.00%	100.00%		100.00%	100.00%	94.00%		100.00%	

Table 28: $PL(j, k)$ under Covering_HLP2_HNRP_a2 model and S=100

$PL(j,k)$		Covering_HLP2_HNRP_a21 S=50																					
$j \backslash k$		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1		100.00		100.00			27.25											100.00				100.00
	2		100.00		100.00					33.59					48.97	35.19			24.23				23.58
	3			41.63	100.00														100.00				
	4		100.00		100.00					59.31			58.62		47.15				100.00	4.04			59.43
	5																						
	6																						
	7		100.00		100.00			2.60							99.70				100.00	0.17	100.00		100.00
	8																						
	9		100.00		100.00	78.26											13.51		76.74		91.18		
	10																						
	11																						
	12		100.00		100.00														100.00				
	13	21.24	100.00		100.00																		
$S_d(k)$	32.00%	46.00%	46.00%	75.00%	92.00%	100.00%	100.00%	100.00%	74.00%	56.00%	56.00%	71.00%	100.00%	100.00%	100.00%	34.00%	15.00%	85.00%	100.00%	100.00%	91.00%	100.00%	100.00%

Table 29: $PL(j,k)$ under Covering_HLP2_HNRP_a21 model and S=50

$PL(j,k)$		Covering_HLP2_HNRP_a21 S=100																					
$j \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1		100.00		100.00			28.41											100.00				89.62	
2		100.00		100.00						60.49				100.00	100.00			13.92				83.02	
3		100.00		100.00										100.00				100.00					
4		100.00		100.00					79.69			82.15						100.00	5.05			100.00	
5																							
6																							
7		100.00		100.00										60.95				100.00		100.00			
8		100.00		100.00														100.00					
9	100.00	100.00		100.00												69.62		100.00		100.00			
10					80.24																		
11		100.00		100.00																			
12		100.00		100.00														100.00					
13	33.38	100.00		100.00																			
$S_d(k)$	73.00%	83.00%		80.00%	94.00%		100.00%	100.00%	100.00%	100.00%	100.00%	100.00%		100.00%	98.00%	77.00%		100.00%	100.00%	94.00%		100.00%	

Table 30: $PL(j,k)$ under Covering_HLP2_HNRP_a21 model and S=100

$PL(j, k)$		Covering_HLP2_HNRP_a22 S=50																					
$j \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1		100.00		100.00														100.00					
2		100.00		100.00					100.00	33.59								16.73				17.53	
3																							
4		100.00		100.00								58.62		66.86				100.00		97.88		100.00	
5																							
6																							
7		100.00		100.00	7.11		5.36		33.51									100.00				100.00	
8																							
9	18.50	100.00		100.00			16.80						18.12		30.37	17.10		61.11	0.58				
10		100.00		100.00												100.00							
11																							
12																							
13																							
$S_d(k)$	32.00%	54.00%		77.00%	8.00%		100.00%		65.00%	56.00%		71.00%		53.00%	30.00%	38.00%		76.00%	80.00%	92.00%		99.00%	

Table 31: $PL(j, k)$ under Covering_HLP2_HNRP_a22 model and S=50

$PL(j, k)$		Covering_HLP2_HNRP_a22 S=100																					
$j \backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1		100.00		100.00														100.00					
2		100.00		100.00					100.00	60.49								100.00				87.66	
3																							
4		100.00		100.00								82.15		89.05				100.00		100.00		100.00	
5																							
6																							
7		100.00		100.00	64.03		14.88		62.05									46.98					
8																							
9	52.37	100.00		100.00			11.68							54.05	64.81	88.13			0.58				
10		100.00		100.00												16.33							
11		100.00		100.00														100.00					
12																							
13																							
$S_d(k)$	91.00%	83.00%		80.00%	75.00%		100.00%		100.00%	100.00%		100.00%		100.00%	63.00%	81.00%		100.00%	80.00%	94.00%		100.00%	

Table 32: $PL(j, k)$ under Covering_HLP2_HNRP_a22 model and S=100

Test problems	p	S=50		S=100	
		y(j)	S_d	y(j)	S_d
Tp1.0	9	1,2,3,4,5,6,7,8,9	100.00%	1,2,3,4,5,6,7,8,9	100.00%
Tp2.0	7	2,3,4,6,7,8,9	96.00%	2,3,4,5,6,7,9	99.00%
Tp3.0	5	2,3,4,7,9	86.90%	2,3,4,7,9	92.10%
Tp4.0	3	3,4,9	64.90%	2,4,9	72.60%
Tp1.0.25	9	1,2,3,4,5,6,7,8,9	100.00%	1,2,3,4,5,6,7,8,9	100.00%
Tp2.0.25	7	2,3,4,6,7,8,9	97.90%	1,2,4,5,6,7,9	100.00%
Tp3.0.25	5	2,3,4,7,9	88.80%	2,4,5,7,9	97.60%
Tp4.0.25	3	3,4,9	71.40%	2,4,9	84.70%
Tp1.0.50	9	1,2,3,4,5,6,7,8,9	100.00%	1,2,3,4,5,6,7,8,9	100.00%
Tp2.0.50	7	2,3,4,6,7,8,9	98.10%	2,3,4,5,6,8,9	100.00%
Tp3.0.50	5	2,3,7,8,9	91.00%	2,3,4,6,9	99.20%
Tp4.0.50	3	2,3,9	78.60%	2,4,9	89.80%

Table 33: Objective function, activated hospitals and S_d values

p=9	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		42.01		82.47		10.52				17.54			77.73							
2	84.01	43.73		55.57		74.61		0.80	38.61			74.67			58.17	50.62				53.12
3	1.40	61.61		94.07		99.48		0.60		95.42		99.03	55.04	47.07	99.06	36.57				
4		50.56		65.31		99.87	25.57		57.54	99.73	7.85	86.31	93.70	35.92				58.10		
5			25.64	0.37																
6				93.41													76.46	83.33		
7	46.05	72.43		83.29					65.04						97.84					
8		72.69		100.00								0.30								
9	50.47	80.49	1.25	78.47	71.17	99.63		0.30		63.53					43.67	97.65			2.02	

Table 34: $PL(j, k)$ with $p = 9$, $S = 100$ Km and $\alpha_j^k = 0$

p=7	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2	84.01	100.00		100.00		85.13		0.80	38.61			74.86			58.17	50.62				53.12
3	1.40	61.61		100.00		99.48		0.60		100.00		99.03	100.00	47.07	99.06	36.57				
4		79.78		100.00		99.87	25.57		57.54	99.73	7.85	86.31	100.00	35.92						
5			25.64	69.61														58.10		
6				100.00													76.46	83.33		
7	46.05	100.00		100.00					65.04						97.84					
8																				
9	50.47	80.49	1.25	100.00	71.17	99.63		0.30		73.15					43.67	97.65			2.02	

Table 35: $PL(j, k)$ with $p = 7$, $S = 100$ Km and $\alpha_j^k = 0$

p=5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2	84.01	100.00		100.00		85.13		0.80	38.61			74.86			58.17	50.62				53.12
3	1.40	61.61		100.00		99.48		0.60		100.00		99.03	100.00	47.07	99.06	36.57				
4		79.78		100.00		99.87	25.57		57.54	99.73	7.85	86.31	100.00	35.92						
5																				
6																				
7	46.05	100.00		100.00					65.04						97.84					
8																				
9	50.47	80.49	51.25	100.00	71.17	99.63		0.30		73.15					43.67	97.65			2.02	

Table 36: $PL(j, k)$ with $p = 5$, $S = 100$ Km and $\alpha_j^k = 0$

p=3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2	100.00	100.00		100.00		100.00		1.80	100.00			100.00			100.00	100.00				53.12
3																				
4		100.00		100.00		100.00	25.57		60.05	100.00	7.85	100.00	100.00	100.00						
5																				
6																				
7																				
8																				
9	51.86	100.00	51.25	100.00	71.17	100.00		0.30		100.00					100.00	100.00			2.02	

Table 37: $PL(j, k)$ with $p = 3$, $S = 100$ Km and $\alpha_j^k = 0$

$p=7$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	62.80			74.78		11.11				76.05			99.83							
2	67.95	93.14		99.96		99.97		0.64	30.89			99.96			99.87	99.75				42.50
3																				
4		40.45		99.98		79.89	20.46		46.03	79.79	6.28	89.91	81.34	85.17						
5			20.51	0.29														46.48		
6				74.73													61.16	66.67		
7	36.84	57.94		82.33					52.03						86.05					
8																				
9	40.37	64.39	1.00	62.78	56.94	79.70		0.72		50.82					99.92	96.00			1.62	

Table 38: $PL(j, k)$ with $p = 7$, $S = 100$ Km and $\alpha_j^k = 0.25$

$p=7$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2	56.00	29.16		37.18		49.74		0.53	25.74			49.78			38.78	33.74				35.41
3	0.93	41.07		62.71		66.32		0.40		63.61		66.02	36.69	31.38	66.04	24.38				
4		65.23		99.97		73.24	17.05		38.36	76.76	5.23	57.54	100.00	23.94						
5																				
6				62.27													50.97	55.56		
7	30.70	48.28		55.53					43.36						65.23					
8		48.46		66.67								0.20								
9	33.64	53.66	0.83	58.82	47.45	66.42		0.20		42.35					29.12	65.10			1.35	

Table 39: $PL(j, k)$ with $p = 7$, $S = 100$ Km and $\alpha_j^k = 0.50$