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# Designing Electricity Tariffs in the Retail Market: A Stochastic Bi-Level Approach

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## 1. Introduction

Over the last decades, the electricity sector has witnessed a fundamental change all around the world. Technological innovation, deployment of renewable generation, new energy policies and changing consumer behavior are all drivers of this challenging transformation. Moreover, the liberalization process has led to the evolution from vertically integrated monopolies towards more competitive energy markets, where retailers are important stakeholders. Operating as intermediaries between end-users and the electricity market, the retailer's business activity is profitable only if the revenue deriving from the electricity selling is higher than the incurred procurement cost. However, the fluctuations in wholesale prices might determine severe financial losses that the retailer should take into account when setting the electricity tariffs. The high competition in the retailer markets makes the pricing problem very involved. On the one hand, the offered rates should be high enough to cover the supply cost and bring an acceptable profit to the retailer, while, on the other hand, they must be low enough to prevent clients from switching energy supplier.

In this paper, we focus on the interaction between a retailer and a class of residential prosumagers, i.e. consumers that also produce energy from renewable sources, such as Photovoltaic (PV) panels. They are also typically equipped with a Battery Electricity Storage (BES) device to efficiently exploit the solar production, mitigating the effects of its intermittent nature. The PV-BES represents a very common energy solution as it has been shown to offer significant economic benefits. Self produced energy, that is not directly used, can be stored in the battery and used later and/or the device can be used to accumulate energy purchased during inexpensive off-peak hours to use when electricity prices are higher. As end-user, the aim of the prosumager is to minimize the electricity bill and the challenge is to exploit the available energy resources to shape the consumption patterns in response to the economic signals. In this way, prosumagers may play an active role in the demand response programs, whose final aim is to favour the change from the traditional "supply follows load" paradigm to the new "load follows supply" one.

We assume that the retailer is interested in designing a dynamic pricing scheme. These structures are expected to become a common practice in smart grids, bringing benefits for all the involved stakeholders: grid operators, retailers and end-users. Differently from the static programs, which are typically contracted for long periods (e.g. one year), the dynamic ones are announced with a short advance, as in our model just one day before the commitment. Within the dynamic pricing schemes, different time-variant options can be considered. According to the Time-of-Use (ToU) scheme, the hours of the day are divided into large blocks (typically, peak, intermediate, and off-peak) and a different rate is charged for each block. In the case of the Critical Peak option (CP), the same tariff is applied for the whole day

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except for some time intervals, where the price is higher. In the case of the Real-Time (RT) pricing, electricity rates vary over short time intervals (one hour or even less). Interested readers are referred to (Dutta and Mitra, 2017) for a review on the main dynamic pricing schemes. In this paper, we focus on the RT scheme, with hourly rates. The other structures can be considered as special cases.

The interaction between the retailer and the prosumager is modeled by a Stackelberg game and the pricing problem is formulated by the Bi-Level (BL) framework (Colson, Marcotte and Savard, 2007). This modeling choice is suggested by the hierarchical nature of the interaction traceable to a leader–follower structure. The retailer, acting as leader, decides first by defining the electricity tariffs. Then, the follower reacts to the communicated prices by defining a new operation management strategy that, fitting the tariff, aims at minimizing the electricity bill. The follower's reaction is fed back to the retailer and affects the actual profit. Therefore, the retailer must consider this kind of interaction when designing the structure of the RT program in order to maximize the profit.

During the last years, increasing attention has been devoted to the development of pricing strategies for electricity retailers considering the relevance of the topic in the liberalized electricity market. The vast majority of the scientific contributions propose mathematical formulations based on the BL framework since it naturally represents the hierarchical structure of the interaction between the energy provider and the customers. Interested readers are referred to (Antunes, Alves and Ecer, 2020) for a comprehensive survey paper. We marginally cite a recent paper by Ferrara, Violi, Beraldi, Carrozzino and Ciano (2021), where the pricing problem is modeled by a different mathematical paradigm.

In the referred contributions, several elements are modeled leading to more and more complex formulations. In (Grimm, Orlinskaya, Schewe, Schmidt and Zöttl, 2020) the authors consider and test different pricing structures, concluding that the RT scheme yields the highest profit for the retailer. A new pricing scheme depending not only on the time, but also on the level of use has been proposed in (Anjos, Brotcorne and Gomez-Herrera, 2021). In (Soares, Alves and Antunes, 2020a), the authors present a deterministic BL model for the definition of ToU tariffs where the lower level problem refers to a consumer with flexible loads. The proposed formulation is further extended and fully tested in (Soares, Alves and Antunes, 2020b). In (Beraldi and Khodaparasti, 2022), the authors analyze the interaction between a retailer equipped with local energy resources and a follower represented by a prosumager.

Even though relevant, the contributions referred above neglect the important issue related to the inherent uncertainty that clearly affects the pricing problem. The pool-based electricity market represents the main source of uncertainty in the retailer's model due to the high price variability. Such uncertainties impose risks that should be carefully considered and properly managed. Moreover, the retailer's profit depends on the follower's reaction that, being also influenced by the weather-related variables, is uncertain too.

To deal with the more involved pricing problem under uncertainty and risk, we propose a stochastic BL model based on the two-stage stochastic programming paradigm. While both deterministic BL and stochastic programming are well established mathematical frameworks, as highlight in Section 2 only a few contributions consider their integration. Following the classical stochastic programming framework, we assume that the uncertain parameters are represented by random variables defined on a given probability space. The nature of the decision process and the structure of the electricity market suggest the adoption of the two-stage paradigm. Here, the decision variables are partitioned in two groups: first and second-stage decisions. While first-stage actions are taken "here-and-now" and should be optimal whatever the future realization of the random variables will be, second-stage decisions are seen as corrective actions that ensure the feasibility of the stochastic constraints. In the proposed model, first-stage decisions refer to the electricity tariffs and to the amount of electricity that the retailer purchases from the Day-Ahead Electricity Market (DAEM), whereas second-stage decisions refer to the amount of electricity exchanged with the Balancing Market (BM) to account for possible imbalances. For a given tariff, the prosumager solves a scenario-based problem to determine the consumption pattern based on the optimal management of the owned energy system.

The injection of uncertainty in the decision process affects not only the constraints, but also the leader's and follower's objective functions requiring a redefinition of the optimality conditions. While for the prosumager we consider the minimization of the expected daily random electricity bill, for the retailer we propose the integration of a safety measure to control the random profit. Variance, mean absolute deviation, expected shortfall are only few examples of risk measures (see, e.g. Aven (2016) for a general review). Here, we consider the $\gamma$-tail mean, a measure aimed to control the expected value of the profit that can be achieved in a given percentage $(1-\gamma)$ of worst case realizations. This measure is known in the literature under different names among which the Conditional-Value-at-Risk (CVaR) is by far the most used one (Rockafellar and Uryasev, 2002), when referring to the loss function. Depending on the choice of the safety level $\gamma$, a different risk attitude can be modeled, thus providing the retailer with a wide range of alternative solutions. Under this respect, the paper contributes to the scientific literature by providing a risk-aware

approach for the design of time-variant pricing schemes. The model is formulated under the general assumption of discrete distribution and the random variables are represented by scenarios, each one occurring with a given probability value. The explicit consideration of the uncertainty (in the deterministic equivalent formulation the recourse variables and the constraints are replicated with the scenarios) increases even further the numerical difficulties of the BL problem posing new computational challenges. To solve the problem, we extend a classical approach used in the deterministic case and based on the derivation of a single level reformulation. We empirically test different solution schemes obtained by combining the approaches used to derive the single level reformulation with those applied to deal with the nonlinearities in the objective function.

To summarize, the major contributions of the paper are given below.

1) From a modeling perspective, we propose a stochastic BL model for the definition of time-variant electricity tariffs that explicitly accounts for the inherent uncertainty affecting the main parameters involved in the decision process and that integrates a safety measure to control the profit variability.

2) Methodologically, we extend the classical approach used in the deterministic setting. We define and test four reformulations obtained by integrating the different approaches used to derive the single level reformulation with those applied to deal with the bilinear terms that appear in the defined model.

3) The paper provides extensive computational experiments carried out a real case study. The numerical performance of the different approaches are tested for an increasing number of scenarios and different safety levels. We validate the significance of explicitly dealing with the uncertainty and the importance of incorporating a safety measure. Managerial insights highlight that risk-adverse retailers seem to achieve on the long run slightly better economic performance.

The rest of the paper is organized as follows. In Section 2, we review the most relevant contributions related to the present work. Section 3 defines the problem and presents the stochastic formulations for the retailer and the prosumager. Section 4 describes the solution approaches, whereas extensive computational experimentation, carried out on a real case study, is discussed in Section 5. Finally, conclusions and future research directions are drawn in Section 6.

## 2. Related scientific literature

Stochastic BL programming has been emerging as an attractive research field since it allows to jointly model two important issues that characterize many real world applications, namely the hierarchical nature of the decision process that may involve different actors with conflicting goals and the uncertainty that naturally conditions the choice of the optimal strategies. However, to the best of our knowledge very few contributions investigate the theoretical properties and propose general solution approaches for such a challenging class of mathematical problems. Among the few, we mention the recent paper by Goshu and Kassa (2020) where the authors focus on stochastic BL programming and propose a hybrid approach to solve a general recourse formulation under specific assumption on the involved leader's and follower's decision variables. The method is tested on small-scale instances taken from the literature. In (Burtscheidt, Claus and Dempe, 2020), the authors study risk-averse BL formulations, involving the expected excess and the mean upper semideviation, for stochastic models where uncertainty affects the right-hand side of the follower problem. No computational results are presented.

From an application standpoint, stochastic BL has been adopted as mathematical paradigm for modeling applications arising in different fields, as network expansion problem (Escudero, Monge and Rodríguez-Chía, 2020), supply chain (Setak, Feizizadeh, Tikani and Ardakani, 2019), to name a few. The number of articles that address the problem of electricity pricing with stochastic BL approaches is still limited, but constantly increasing. However, typically the actors involved in the interaction are different from those considered in our contribution.

In (Askeland, Burandt and Gabriel, 2020) the authors propose a stochastic BL model where the upper level problem refers to a distribution system operator that is responsible for building and maintaining the electricity grid. The aim is to minimize the total cost related to the network loss and load curtailment. The lower level is represented by a prosumer equipped with a system of PV panels, who aims to minimize the procurement costs. Uncertainty related to market price, solar production, and load is represented by means of a scenario set.

Dadashi, Haghifam, Zare, Haghifam and Abapour (2020) study the interaction between an electricity retailer and a Demand Response Aggregator (DRA), a new entity responsible for aggregating scattered distributed energy resources through appropriate price-based incentives. A two-stage stochastic BL programming approach is proposed to address

the short-term decision making problem faced by the retailer. In particular, the retailer acts as the leader and aims at maximizing the profit through selling energy to clients, exchanging energy in the wholesale market, as well as making a hierarchical connection with the DRA. On the contrary, the DRA, as the follower, tends to maximize its profit via making interaction with the retailer and customers.

In a working paper, Brotcorne, Lepaul and von Niederhäusern (2021) propose a rolling horizon approach for a BL stochastic pricing problem where the leader is an energy supplier and the follower is a smart grid operator that has contract with consumers and may decide on their consumption patterns. The unknown information is part of the lower level that is dealt as a multi-stage problem, whereas the upper level, differently from our case, is deterministic.

A stochastic BL model for a short-term decision making problem faced by an electricity retailer participating in a liberalized Distributed Renewable Energy (DRE) market has been proposed in (Do Prado and Qiao, 2020). Differently than in our case, the DRE producers represent the lower level decision makers. In the upper level, the retailer minimizes the weighted sum of the expected procurement cost and the CVaR risk measure. The upper level problem is formulated as a two-stage model where the first-stage decisions refer to the amount of electricity purchased in the DAEM and the DRE price bids, whereas second-stage decisions refer to the involvement in the BM. At the lower level, each DRE agent reacts to the prices offered by the retailer determining the percentage of share to be supplied with the objective of maximizing the total revenue. A single-level reformulation of the model is proposed and tested.

The retailer-consumer interaction has been, for example, considered in (Zugno, Morales, Pinson and Madsen, 2013) and (Sekizaki, Nishizaki et al., 2019). In the first contribution, the authors propose a stochastic three-stage BL formulation to differentiate between factors being revealed before real-time operations and those disclosed on an ex-post basis. The dynamic tariffs charged to the end-users are scenario-dependent and the expected profit is maximized. The problem is reformulated as a single level mixed integer problem using the Karush–Kuhn–Tucker (KKT) conditions which are linearized. In the other contribution, the authors propose a stochastic BL model for the ToU price settings considering a medium-term planning horizon. The retailer is assumed to procure energy by forward contracts and/or from the DAEM, whereas the consumer is assumed to be equipped with a storage device. Uncertainty affects the electricity market price and the consumer's demand and the preference of the retailer is reflected by employing a fractile criterion to maximize using a stochastic aspiration level. The considered measure actually corresponds to the Value-at-Risk that, unlike the CVaR we apply in this paper, is known to not be a coherent risk measure.

Our paper contributes to the scientific literature by proposing a stochastic BL approach to support the retailer in the definition of daily RT pricing schemes. The follower problem refers to a prosumager, not just a consumer, who reacts to the offered tariffs by optimizing the management of the available resources. At the upper level, the retailer has the possibility to calibrate the pricing strategy taking into account his/her risk attitude by properly setting the safety level.

## 3. Problem definition and formulation

We consider the problem faced by an energy retailer who wants to define a dynamic pricing scheme with time-differentiated rates. The tariffs are communicated the day before and are daily computed running the proposed model with new realizations of the uncertain data that are generated, each time, using updated information.

The electricity tariffs should be competitive with respect to those offered by other retailers in the market, but high enough to repay the risk that the retailer takes when entering into a contract with a prosumager. Different sources of uncertainty concur to increase the retailer financial risk: market prices that are unknown when the retailer defines the rates, weather-related variables that influence the production from renewable energy resources and, thus, the prosumager's demand and the retailer's revenue.

We represent these uncertain parameters as discrete random variables defined on a given probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and we denote by $\mathcal{S}= \{1, \dots, S\}$ the set of realizations (scenarios) and by $\pi_s$ the corresponding probability of occurrence. The choice of considering discrete random variables is general and it is appropriate when, as in our case, a considerable amount of historical observations are readily available.

In the proposed model, the upper level problem is represented as a two-stage problem with recourse. This modeling choice is suggested by the structure of the electricity market that naturally fits the recourse paradigm and by the retailer's decision process. In particular, the retailer makes first-stage decisions about the electricity rates and the amount of electricity to purchase in the DAEM. Such decisions are taken in the face of uncertainty and should be optimal regardless of the realization of the uncertain parameters. The scenario-dependent corrective actions are represented by the amount of electricity exchanged with the BM to guarantee the satisfaction of prosumager's demand. Given the electricity tariff, the follower determines the residual demand by solving a scenario-based problem aimed at defining

the optimal management of the local energy system that minimizes the daily electricity bill. The prosumager's reaction is fed back to the retailer and affects the actual profit. Figure 1 shows the general structure of the proposed stochastic BL model.

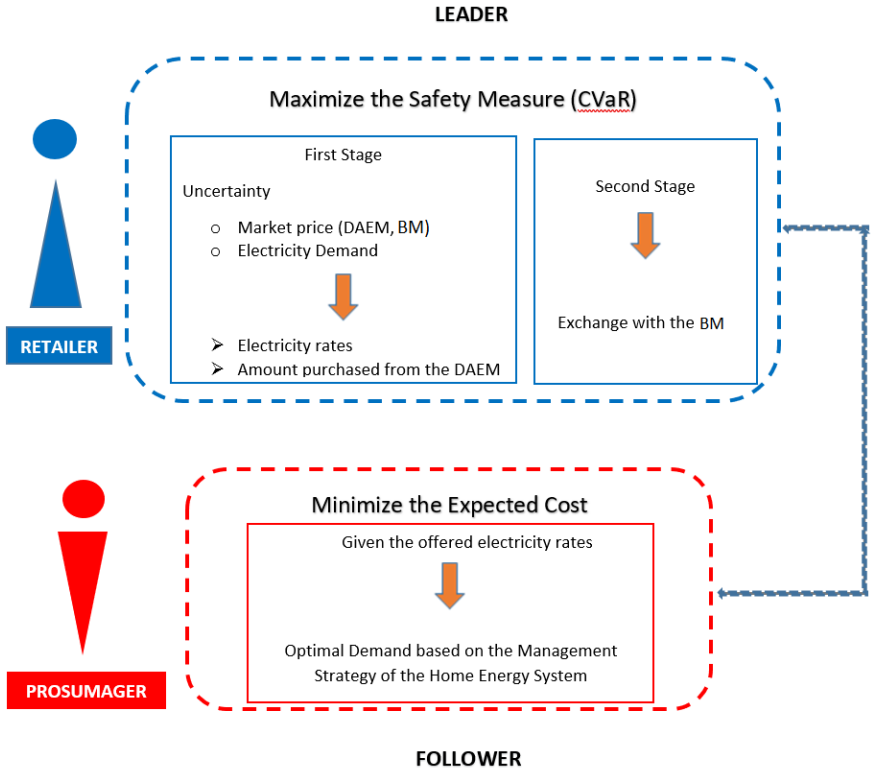



**Figure 1:** Structure of the proposed stochastic BL model

The presentation of the upper and lower level problem is reported below. For the sake of clarity, we list the sets, parameters and decision variables used in the proposed formulation. Acronyms UL and LL are used to distinguish the decision variables referring to the retailer and prosumager, respectively.

Sets and indices:

| | |
|---|---|
| $\mathcal{T} = \{1, \cdots, T\}$ | Set of time slots indexed by $t$ |
| $\mathcal{S} = \{1, \cdots, S\}$ | Set of scenarios indexed by $s$ |

Parameters:

| | |
|---|---|
| $\alpha_t$, $\beta_t$ | Lower and upper bound for the electricity tariff at time $t$ |
| $\Gamma$ | Bound on the offered average tariff |
| $C$ | Nominal capacity of the BES |
| $\mu_1$, $\mu_2$ | Operation range of the BES |
| $\tau_c$, $\tau_d$ | Charging and discharging rates |
| $\eta_c$, $\eta_d$ | Efficiency rates for energy charge and discharge |
| $soc_0$ | Initial state of charge |
| $cb$ | Cost of charging and discharging operations |
| $\pi_s$ | probability occurrence of scenario $s$ |
| $DAP_{ts}$ | The DAEM price at time $t$ under scenario $s$ |
| $BP^+_{ts}$ | The purchasing BM price at time $t$ under scenario $s$ |
| $BP^-_{ts}$ | The selling BM price at time $t$ under scenario $s$ |
| $\psi_{ts}$ | The PV energy production at time $t$ under scenario $s$ |
| $b_{ts}$ | Base load at time $t$ under scenario $s$ |
| $\varsigma$ | Upper bound on the prosumager's demand defined by contract |
| $\gamma$ | Safety level |

Decision variables:

| | |
|---|---|
| $r_t$ | Electricity rate at time $t$ (UL) |
| $x_t$ | Energy amount purchased from the DAEM at time $t$ (UL) |
| $\Delta^+_{ts}$ | Shortage in supply at time $t$ under scenario $s$ (UL) |
| $\Delta^-_{ts}$ | Excess supply at time $t$ under scenario $s$ (UL) |
| $\phi_s$ | The retailer's profit under scenario $s$ (UL) |
| $\upsilon_s$ | Auxiliary scenario-dependent variable (UL) |
| $\xi$ | Auxiliary variable (UL ) |
| $d_{ts}$ | Follower's residual demand at time $t$ under scenario $s$ (LL) |
| $soc_{ts}$ | State of charge for the BES at time $t$ under scenario $s$ (LL) |
| $e^c_{ts}$ | Energy charged into the BES at time $t$ under scenario $s$ (LL) |
| $e^d_{ts}$ | Energy discharged from the BES at time $t$ under scenario $s$ (LL) |

### 3.1. The retailer problem

The retailer problem is formulated as a two-stage stochastic programming model. The first group of first-stage decisions refer to the offered electricity rates, denoted by $r_t$. To account for the market competitiveness, the rates are properly bounded, by specifying the minimum, the maximum, and the average value that could be reached during a day, as imposed by the set of constraints (1)-(2):

$$\alpha_t \leq r_t \leq \beta_t \quad \forall t \in \mathcal{T} \tag{1}$$

$$\frac{1}{T}\sum_{t=1}^{T} r_t \leq \Gamma \tag{2}$$

We remind that our model does not explicitly consider the possibility that the prosumager might switch to a different retailer, since the switching process is rather low and involves a time horizon different from that considered to set dynamic daily prices.

To cover the follower's demand, the retailer defines the procurement plan in terms of amount to purchase from the wholesale electricity market. In particular, the quantity to purchase from the DAEM, denoted by $x_t$, should be decided in advance for all the periods of the upcoming day, before knowing the cleared prices. These decisions are

determined taking into account an estimate of the consumption pattern that the prosumager is supposed to provide or that the retailer may work out based on historical data. However, since the residual prosumager request depends on weather-related variables (e.g., the solar irradiance influences the electricity production), corrective actions could be taken to ensure the supply-demand balance. Let $d_{ts}$ denote the amount of electricity required by the follower at time $t$ under scenario $s$; such variables are under the follower's control, who determines their optimal values by solving the lower level problem for a given set of offered rates. The second-stage upper level variables are defined in the retailer problem to guarantee the satisfaction of the stochastic demand constraints. In particular, for each time period $t$ and under every scenario $s$, we define the variables $\Delta_{ts}$ to account for the mismatch $(d_{ts} - x_t)$ between the actual follower's demand and the DAEM supply. Two situations may occur:

- $\Delta_{ts} > 0$ : in this case the demand is higher than amount procured from the DAEM and the missing power should be purchased from the BM at higher prices
- $\Delta_{ts} < 0$: the retailer has overestimated the demand and the residual amount should be fed back to the power grid

The corrective actions are denoted by $\Delta^+_{ts}$ accounting for the missing power and defined as $[d_{ts} - x_t]_+$, and by $\Delta^-_{ts}$ accounting for the surplus and defined as $[d_{ts} - x_t]_-$. The corresponding prices are denoted by $BP^+_{ts}$ and $BP^-_{ts}$, respectively, and are typically higher (for purchasing) and lower (for selling) than the DAEM prices.
The following set of constraints express the relation between first and second-stage variables. In particular, (3)-(4) define the corrective actions, whereas constraints (5) refer to satisfaction of the prosumager's demand.

$$\Delta^+_{ts} \geq d_{ts} - x_t \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{3}$$
$$\Delta^-_{ts} \geq x_t - d_{ts} \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{4}$$
$$d_{ts} = x_t + \Delta^+_{ts} - \Delta^-_{ts} \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{5}$$
$$\Delta^+_{ts},\ \Delta^-_{ts} \geq 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{6}$$

We assume that the retailer is risk-averse and we model this attitude by introducing as objective function a safety measure. In particular, we consider the $\gamma$-tail mean by which we control the expected profit that can be gained in a given percentage of worst case realizations. We also refer to this measure as the CVaR, name typically used when considering random losses (Rockafellar and Uryasev, 2000).

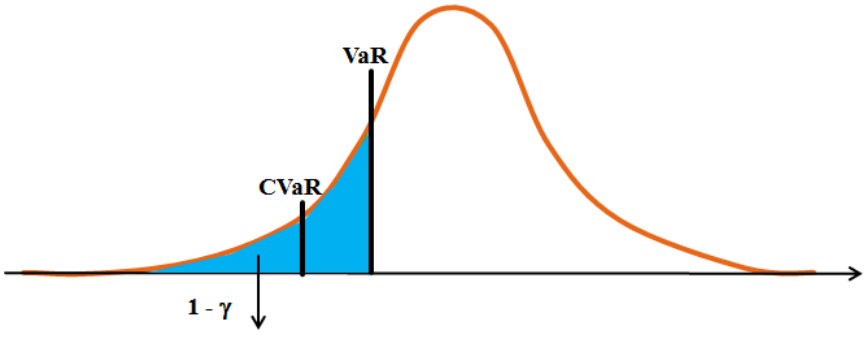



**Figure 2:** Probability density function of the retailer's profit

Figure 2 shows a representation of the CVaR. Focusing on the left tail of the profit function, the aim is to maximize the expected value computed on the (1-$\gamma$) percentage of worst case realizations. Let $\phi_s$ denote the retailer's profit under scenario $s$ as defined in (7):

$$\phi_s = \sum_{t=1}^{T} r_t\, d_{ts} - \sum_{t=1}^{T} DAP_{ts} x_t - \sum_{t=1}^{T} BP^+_{ts}\Delta^+_{ts} + \sum_{t=1}^{T} BP^-_{ts}\Delta^-_{ts} \qquad \forall s \in \mathcal{S} \tag{7}$$

Here the first term represents the total revenue gained from selling energy to the prosumager, whereas the second term displays the cost of the energy purchase from the DAEM and the third and fourth terms account for the balancing

actions. Following the results presented in (Rockafellar and Uryasev, 2002), we write the retailer objective function, also denoted in the following with $\text{CVaR}_\gamma$, as:

$$\max z_U = \max \left( \xi - \frac{1}{1-\gamma} \sum_{s=1}^{S} \pi_s [\xi - \phi_s]_+ \right) \tag{8}$$

Here $\xi$ is a free decision variable, representing the VaR and $[\xi - \phi_s]_+ = \max(\xi - \phi_s, 0)$. A linear reformulation can be easily obtained by introducing supporting decision variables $v_s$ defined as:

$$v_s \geq \xi - \phi_s \quad \forall s \in \mathcal{S} \tag{9}$$

$$v_s \geq 0 \quad \forall s \in \mathcal{S} \tag{10}$$

It is easy to recognize that by varying the safety level $\gamma$ between 0 and 1, different risk attitudes can be represented. The two extreme cases refer to $\gamma$ approaching to 0 and 1. In case of $\gamma = 0$, no safety margin is imposed reflecting a risk-neutral position in which the retailer is not concerned about risk and only aims at maximizing the expected profit. In case of $\gamma \to 1$, e.g. $\gamma = 0.9999$, the retailer is extremely risk-averse and seeks the profit maximization under the worst case. Clearly, such an attitude towards risk might provide over-conservative solutions and mild positions represented by suitable setting of $\gamma$ could be more preferable.

### 3.2. The prosumager problem

The prosumager reacts to the announced electricity tariffs by optimally defining the power consumption and exploiting the owned energy system. The weather conditions clearly influence the solar production and, thus, the residual demand. The presence of the BES allows to mitigate uncertainty providing scenario-dependent consumption patterns. In the proposed model, all the lower level decision variables have a "wait-and-see" nature. In particular, we denote by $soc_{ts}$ the state of charge at time $t$ and under scenario $s$, whereas $e^c_{ts}$ and $e^d_{ts}$ refer to the amount of power charged to and discharged from the storage device, respectively.

Constraints (11) represent the balancing equation used to relate the state of charge over two consecutive periods with the amount of energy charged and discharged:

$$soc_{ts} = soc_{t-1s} + \eta_c \, e^c_{ts} - \frac{1}{\eta_d} e^d_{ts} \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{11}$$

where parameters $\eta_c$ and $\eta_d$ account for the energy-loss in the storage operations. We note that for $t = 1$, the state of charge is equal to a known value denoted by $soc_0$ and we impose a continuity constraint by requiring that the same amount of energy should be present in the battery at the end of the planning horizon:

$$soc_{Ts} = soc_0 \quad \forall s \in \mathcal{S} \tag{12}$$

The set of constraints in (13)-(14) impose some upper bounds on the total amount of power charged and discharged. Moreover, constraints (15)-(16) ensure that the amount of energy in the BES is always within a minimum and a maximum level, defined as function of the nominal capacity $C$.

$$e^c_{ts} \leq \tau_c C \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{13}$$

$$e^d_{ts} \leq \tau_d C \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{14}$$

$$soc_{ts} \geq \mu_1 C \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{15}$$

$$soc_{ts} \leq \mu_2 C \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{16}$$

For each time $t$ and under each scenario $s$, the residual demand is expressed as:

$$d_{ts} = b_{ts} - \psi_{ts} - e^d_{ts} + e^c_{ts} \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{17}$$

where $\psi_{ts}$ and $b_{ts}$ represent, respectively, the total PV energy production and the base load at time $t$ under scenario $s$. We remind that the amount of electricity the prosumager can purchase from the retailer during each time period is limited by contract as expressed by constraint (18):

$$d_{ts} \leq \varsigma \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} \tag{18}$$

The aim of the prosumager is to minimize the electricity bill. Under each scenario a different cost can be incurred. Keeping a risk-neutral position towards risk, the objective function can be written as:

$$\min z_L = \sum_{s=1}^{S} \pi_s \sum_{t=1}^{T} \left( r_t \, d_{ts} + cb \, e^c_{ts} + cb \, e^d_{ts} \right) \tag{19}$$

where the last two terms represent the cost associated with the charging and discharging operations. We note that, as shown in (Garifi, Baker, Christensen and Touri, 2018), the presence of such costs makes the solutions with simultaneous charging and discharging be suboptimal, allowing to avoid the introduction of binary variables and the related constraints.

We point out that the proposed BL formulation can be extended to address the more general multi-follower case in which different classes of end-users are considered. In the proposed model, we have implicitly assumed that all the followers are identical and are represented by a reference prosumager so that the game plays out on an individual level. Under the more general multi-follower case, different rates may be offered to the followers and different LL problems may be defined to represent specific restrictions. This would exacerbate the computational complexity of the problem that is already quite high, even for the deterministic case.

## 4. The solution approach

The presence of uncertain parameters increases further the complexity of a problem already difficult to solve in the deterministic setting. Indeed, finding the global solution of a deterministic BL problem is a great challenge because of the NP-hardness of this class of problems.

The proposed solution approach extends a classical methodology used in the deterministic setting based on the idea of deriving a single level reformulation. Even though the size of the resulting model increases with the number of scenarios, the problem exhibits a nice structure that can be exploited in the solution process. We may observe that for a given upper level first-stage solution $r_t$, the lower level problem (11)-(19) results in a linear programming problem that can be decomposed by scenarios. As usual in the deterministic case, we may work out with the corresponding dual, that is defined as:

$$\begin{aligned}
\max w = \quad & \sum_{s=1}^{S} \sum_{t=1}^{T} \left( (b_{ts} - \psi_{ts}) \delta_{ts} - \tau_c C \lambda^c_{ts} - \tau_d C \lambda^d_{ts} + \mu_1 C \lambda^{soc}_{ts} - \mu_2 C \lambda^{'soc}_{ts} - \varsigma v_{ts} \right) + && (20) \\
& \sum_{s=1}^{S} soc_0 (\gamma_{1s} + \gamma^{'}_{s}) \\
& \delta_{ts} - v_{ts} \leq \pi_s r_t \qquad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} && (21) \\
& \delta_{ts} + \frac{1}{\eta_d} \gamma_{ts} - \lambda^d_{ts} \leq \pi_s cb \qquad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} && (22) \\
& -\delta_{ts} - \eta_c \gamma_{ts} - \lambda^c_{ts} \leq \pi_s cb \qquad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} && (23) \\
& \gamma_{ts} - \gamma_{t+1s} + \lambda^{soc}_{ts} - \lambda^{'soc}_{ts} = 0 \qquad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} && (24) \\
& \gamma_{Ts} + \gamma^{'}_{s} + \lambda^{soc}_{Ts} - \lambda^{'soc}_{Ts} = 0 \qquad \forall s \in \mathcal{S} && (25) \\
& \delta_{ts}, \gamma_{ts}, \gamma^{'}_{s} \qquad free \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} && (26) \\
& \lambda^c_{ts}, \lambda^d_{ts}, \lambda^{soc}_{ts}, \lambda^{'soc}_{ts}, v_{ts} \geq 0 \quad \forall t \in \mathcal{T} \; \forall s \in \mathcal{S} && (27)
\end{aligned}$$

where $\gamma_{ts}, \gamma^{'}_{s}, \lambda^c_{ts}, \lambda^d_{ts}, \lambda^{soc}_{ts}, \lambda^{'soc}_{ts}, \delta_{ts}$ and $v_{ts}$ are the dual variables associated with constraints (11)-(18), respectively. We may observe that the follower's problem is always feasible and bounded. Thus, on the basis of the strong duality theorem both the primal and the dual problems have optimal solutions and the corresponding objective function values are equal.

Two main approaches can be used to derive a single level reformulation. The first one, referred in the following with the acronym CS, relies on the idea of replacing the follower's problem by its KKT conditions (Zare, Borrero, Zeng and Prokopyev, 2019). The equivalence of the KKT formulation and the prosumager's problem is guaranteed by the linearity of the latter for fixed upper level $r_t$ solutions. The complementary slackness conditions associated with

the inequality constraints are represented by the following two set of constraints, (28)-(32) for the primal problem and (33)-(23) for the dual one:

$$(\tau_c C - e^c_{ts})\lambda^c_{ts} = 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{28}$$
$$(\tau_d C - e^d_{ts})\lambda^d_{ts} = 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{29}$$
$$(soc_{ts} - \mu_1 C)\lambda^{soc}_{ts} = 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{30}$$
$$(\mu_2 C - soc_{ts})\lambda'^{soc}_{ts} = 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{31}$$
$$(\varsigma - d_{ts})v_{ts} = 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{32}$$
$$(\pi_s r_t - \delta_{ts} + v_{ts})d_{ts} = 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{33}$$
$$(\pi_s cb - \delta_{ts} - \frac{1}{\eta_d}\gamma_{ts} + \lambda^d_{ts})e^d_{ts} = 0 \quad \forall t\ \ \forall s \tag{34}$$
$$(\pi_s cb + \delta_{ts} + \eta_c\gamma_{ts} + \lambda^c_{ts})e^c_{ts} = 0 \quad \forall t\ \ \forall s \tag{35}$$

We underline that the system of KKT conditions (11)-(18), (21)-(23) is linear, with the exception of the complementary slackness conditions that can be linearized by adopting, for example, a Big-M approach, that, as known, entails the introduction of binary variables and the selection of suitable big-M constants (see, Kleinert, Labbé, Plein and Schmidt (2020)). While the structure of the problem can be easily exploited in determining proper values for these constants, the number of binary variables to introduce becomes prohibitive when considering a sufficiently large number of scenarios. Indeed, it is equal to $|T||S|(np + nd)$ where $np$ and $nd$ denote the number of primal and dual variables, respectively.

An alternative method relies on the application of the strong duality theorem. In the approach, referred in the following with the acronym SD, the constraint (36) equating the primal and dual objective function of the follower's problem, is used in place of (28)-(23):

$$\sum_{s=1}^{S}\sum_{t=1}^{T}\left((b_{ts} - \psi_{ts})\delta_{ts} - \tau_c C\lambda^c_{ts} - \tau_d C\lambda^d_{ts} + \mu_1 C\lambda^{soc}_{ts} - \mu_2 C\lambda'^{soc}_{ts} - \varsigma v_{ts}\right) + \tag{36}$$
$$\sum_{s=1}^{S} soc_0(\gamma_{1s} + \gamma'_s) = \sum_{s=1}^{S}\pi_s\sum_{t=1}^{T}(r_t\, d_{ts} + cb\, e^c_{ts} + cb\, e^d_{ts})$$

Common to both the approaches used to derive the single level reformulation, there is the issue related to the presence of bilinear terms ($r_t\ d_{ts}$) that appear in the constraint (36). We have considered two different strategies. The first one, referred as BE, is based on the binary expansion of the variables $r_t$. The meaning of such variables and the presence of box constraints, would encourage this choice. Assuming that one cent of increment can be considered, $r_t$ could be rewritten as:

$$r_t = \alpha_t + \sum_{k=1}^{K} kl_{kt}$$

where $K = \lceil\frac{\beta_t - \alpha_t}{1}\rceil$ and $l_{kt}$ are binary variables such that $\sum_{k=1}^{K} l_{kt} \leq 1$. Besides the variable $l_{kt}$, that are $|T|K$, additional variables and constraints are then required to linearize the product of the variables $l_{kt}$ and $d_{ts}$. Depending on the number of scenarios, the computational effort can be really high as confirmed by the computational experiments.

The second strategy, referred as DR, relies on the application of the dual reformulation, that as shown in (Costa, Ng and Foo, 2017) typically provides better approximation with respect to the primal reformulation, which is, in fact, based on the McCormick's inequalities. In particular, we introduce the auxiliary variables $w_{ts} = r_t\, d_{ts}$ and the following set of constraints:

$$r_t = (\lambda^1_{ts} + \lambda^2_{ts})\,\alpha_t + (\lambda^3_{ts} + \lambda^4_{ts})\,\beta_t \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{37}$$
$$d_{ts} = (\lambda^1_{ts} + \lambda^3_{ts})\,\theta + (\lambda^2_{ts} + \lambda^4_{ts})\,\varsigma \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{38}$$
$$w_{ts} = \lambda^1_{ts}\,\alpha_t\,\theta + \lambda^2_{ts}\,\alpha_t\,\varsigma + \lambda^3_{ts}\,\beta_t\theta + \lambda^4_{ts}\,\beta_t\,\varsigma \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{39}$$
$$\sum_{j=1}^{4}\lambda^j_{ts} = 1 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S} \tag{40}$$

$$\lambda^{j}_{ts} \geq 0 \qquad \forall t \in \mathcal{T}\ \forall s \in \mathcal{S}\ j = 1, \ldots, 4 \tag{41}$$

where the lower and upper bounds for both variables are known in advance. In particular, for $r_t$, the bounds are defined by (1) whereas for $d_{ts}$ the upper bound $\varsigma$ is defined by contract and the lower bound $\theta$ is typically set to 0. We remind that the solution of the reformulated problem provides a global maximum for the relaxed convex problem and thus an upper bound for the original problem. A lower bound can be obtained by solving the original non-convex problem using the values obtained from the relaxed problem and then checking for feasibility.

By combing the two approaches used to derive the single level reformulation, i.e. CS, SD, with those adopted to deal with the bilinear terms, i.e. BE, DR, four different solution schemes have been derived and empirically tested.

## 5. Computational experiments

This section reports on the computational experiments carried out on a real case study to assess the effectiveness of the proposed approach and draws a number of managerial insights. The model and the solution algorithms are coded in GAMS 24.4.6 and solved using ILOG CPLEX (Bussieck and Meeraus, 2007). All the experiments have been performed on an Intel®Core i7 2.6 GHz, with 16.0 GB of RAM memory.

### 5.1. Case study and data setting

In the considered case study, the leader is represented by a retailer operating in the Italian electricity market and the follower by a group of residential prosumagers which are assumed to be identical in terms of available energy system, demand, and so on. Under this assumption, the leader-follower game plays out on a individual level. The extension to the more general multi-follower case, where different rates can be defined for each class of prosumer, is quite straightforward, but involves an additional increase in the computational time. We point out that the solution of the single follower version provides an upper bound on the risk for the multi-follower case and therefore a lower bound on the optimal profit if an equivalent risk measure is used.

The reference prosumager is representative of a large family (with 5 members) resident in Southern Italy equipped with a local PV-BES system that partially cover the electricity demand. The data used in the experiments have been collected within an Italian funded research project[1]. The technical details of the installed technologies are shown in Table 4 in the Appendix.

It is assumed that the prosumager cannot sell back the energy produced in excess to the demand. The extension of the proposed model to this case, which involves the definition of follower's selling rates, is the subject of an ongoing research. In the simulation, we have assumed that the PV system can guarantee partial coverage of the base load. The rate of self-sufficiency is naturally affected by the meteorological conditions and especially by solar irradiance, which are subject to uncertainty. Details concerning the daily consumption pattern and energy production from PV system are reported in Figures 12 and 13 in the Appendix.

The retailer mainly procures electricity from the DAEM and uses the BM to account for imbalance. It is assumed that the retailer is a price-taker with no possibility to influence the market prices. According to the organization of the Italian market[2], the DAEM prices, labeled as National Single prices, are differentiated by hours and cleared the day before delivery (Beraldi, Conforti, Triki and Violi (2004)). Figure 3 shows the DAEM prices for some working days in winter (January-March 2020). We may notice that prices follow a trend with higher rates during peak-hours. However, for the same hour, a high variation can be appreciated over different days, confirming the importance to explicitly account for uncertainty in pricing problems.

We recall that the retailer is interested in defining dynamic pricing scheme with hour-dependent rates. In particular, the rates are announced the day before the delivery and are computed by solving each time a different instance of the proposed formulation obtained by using more recent and more reliable information. Weather forecasts become more and more accurate over time, whereas the cleared DAEM prices are useful to generate scenario prices for the following day.

As in any stochastic programming problem, the scenario generation represents a critical issue since the quality of generated realizations affects the robustness of the provided recommendations. The relevance of the topic has motivated the considerable number of publications appeared during the last decades. We refer the interested readers to (Mitra and Domenica, 2010) for an overview on the main approaches. Reduction techniques (see, for example, (Heitsch and

[1] COMESTO: Community Energy Storage

[2] https://www.mercatoelettrico.org/it/

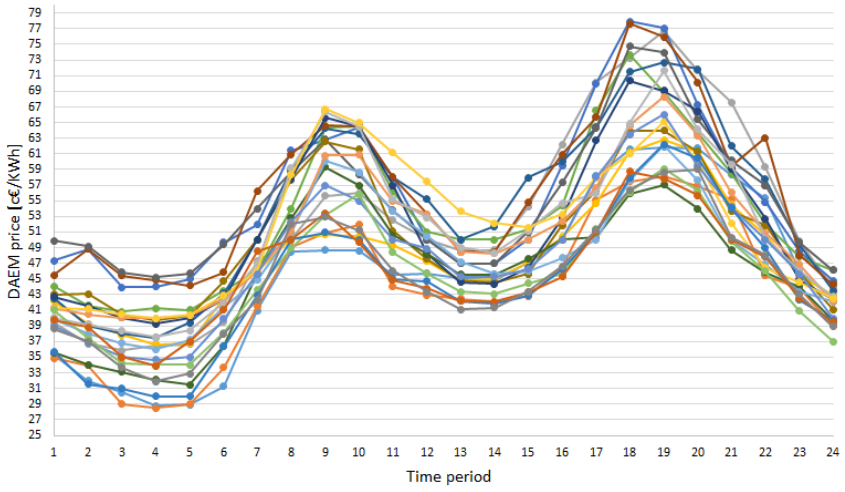



**Figure 3:** The DAEM prices for weekdays in winter

Römisch, 2003)) are often used to shrink the size of the generated scenario set, provided that in-sample and out-of-sample stability properties are ensured. In the following, we detail the method applied to generate the scenarios used in our numerical experiments.

### *5.1.1. Scenario generation*

In the proposed formulation, the scenario prices have been generated by using a mean reverting model similar to the one applied in (Beraldi, Violi, Scordino and Sorrentino, 2011) and widely used in the energy sector. As other energy commodities, spot prices fluctuate around an equilibrium value. When the price of a commodity is low, its supply is likely to decline and its demand to expand, lifting the level of prices. On the contrary, when the spot price is particularly high, the supply tends to increase and the demand to decrease, pushing the level of prices down. The mean reverting model incorporates the tendency of energy prices to gravitate towards a "normal" equilibrium price level, which is usually governed by a set of heterogeneous issues. In the proposed model, prices refer to both the DAEM and the BM. We have generated scenarios for the DAEM prices, $DAP_{ts}$, and we have assumed that those for the BM are 20% higher for the purchasing prices $BP^{+}_{ts}$ and 20% lower for the selling prices $BP^{-}_{ts}$, respectively.

Preliminary to the application of the Monte Carlo simulation technique used to generate the scenario prices, is the statistical analysis that we have carried out on a large set of historical data collected within two years before the period used in the simulation. Obviously, the difference between working and weekend days has been considered in the derivation of the statistical data used in the model. In particular, we have applied the formula reported below to generate the prices for the subsequent day starting from the known data of the previous day:

$$DAP_{ts} = \widehat{DAP}_t + \theta_t(\overline{DAP}_t - \widehat{DAP}_t) + \chi_t \epsilon_{ts} \tag{42}$$

Here $\widehat{DAP}_t$ represents the DAEM price for the same hour $t$ of the day before, the term $\theta_t(\overline{DAP}_t - \widehat{DAP}_t)$ represents a mean reverting component with $\overline{DAP}_t$ denoting a long run equilibrium price, assumed to be equal to the average value for hour $t$ computed by considering the historical data. The parameters $\theta_t$ and $\chi_t$ are estimated by using the linear regression technique. Finally, the random shock $\epsilon_{ts}$ is obtained by considering the extraction from a standard normal distribution $\mathcal{N}(0, 1)$.

Besides the electricity prices, scenarios have been generated to account for uncertainty in the energy production. We remind that the PV production is influenced by the solar irradiance and thus by the meteorological conditions (see, for example, (Algieri, Beraldi, Pagnotta and Spadafora, 2021) for more details). Scenarios for PV production, denoted in the model as $\psi_{ts}$, have been generated by sampling from a normal distribution, with a given mean $\psi_t$ and standard deviation $\sigma_t$. Data used in the simulation have been taken from the paper of Di Somma, Graditi, Heydarian-Forushani, Shafie-khah and Siano (2018). A similar approach has been used to generate the scenarios, $b_{ts}$, for the base load.

| Scenarios | SD-DR | CS-DR | SD-BE | CS-BE |
|---|---|---|---|---|
| 1 | 0.12 | 1.23 | 1.81 | 3.27 |
| 5 | 0.15 | 20.55 | 39.4 | 200.67 |
| 10 | 0.51 | 45.88 | 61.23 | 2500.12 |
| 20 | 1.01 | 1000.12 | 1141.21 | - |
| 50 | 2.44 | - | - | - |

**Table 1**
Solution times (in seconds) for different configurations

| | $\gamma$ | | |
|---|---|---|---|
| Scenarios | 0 | 0.5 | 0.9999 |
| 50 | 1.46 | 2.55 | 2.075 |
| 200 | 18.00 | 25.730 | 24.20 |
| 500 | 113.03 | 187.72 | 206.77 |
| 1000 | 1251.01 | 2252.70 | 2805.75 |

**Table 2**
Solution time of the SD-DR as a function of the scenario number and the safety level

## 5.2. Computational performance

As highlighted in Section 4, different solution approaches can be designed by combining the strategy used to derive the single level reformulation, CS versus SD, with that applied to deal with the bilinear terms, BE versus DR. By combining any pair of such options, we have derived four different configurations. Table 1 shows the solution times (in seconds) for each configuration for an increasing number of scenarios. We note that the reported values refer to the average solution time obtained when solving the problem with different safety levels $\gamma \in [0, 1)$ with a step increment of 0.1. In the Table, we report a "-" for the cases for which no feasible solution is determined within a time limit of 2700 seconds.

Looking at the results, we may observe that the solution time increases with the number of scenarios and for some combinations even with a limited number, the time limit is exceeded. For example, the combination CS-BE seems to be the most unfavorable one in terms of solution time, requiring more than 40 minutes to solve an instance with only 10 scenarios. This result can be explained by observing that, as expected, the introduction of binary variables required for all the approaches, but the SD-DR, negatively impacts on the computational performance. In the case of the CS-BE approach, binary variables (and additional constraints) are introduced to linearize the complementary slackness conditions and the terms used in the binary expansion. While in the deterministic case, corresponding to the solution with only one scenario, the difference in the computational time is very low, for the stochastic case, especially when considering a sufficiently high number of realizations, the computational burden is more evident.

On the basis of the numerical evidence, we have adopted the SD-DR as solution approach in our numerical experiments. Table 2 reports the solution times on larger instances from 50 up to 1000 scenarios. We have considered three safety levels corresponding to different positions towards risk. In the following, we shall use the label "neutral" to denote risk-neutrality represented by a value $\gamma$ equal to 0, "moderate" for a moderate position, with $\gamma = 0.5$ and, finally, the label "extreme" for the highest risk aversion, associated with $\gamma$ equal to 0.9999. We may notice that in all the tested instances, the lowest computational effort corresponds to the case of $\gamma$ equal to 0. We remind that in this case, the retailer aims at maximizing the expected profit and she is not concerned about potential losses that can be experienced.

Analyzing the results, we may also note the increase in the solution times with the scenario number. For example, if we consider the average solution time (over different $\gamma$ values), we may notice that the time required to solve the problem with 500 scenarios is about 8% of the time required for a doubled number of realizations. Based on our experiments, for larger scenario sets, the problem becomes less tractable; for example for the case with 1500 scenarios, the computational time was above 3600 seconds.

## 5.3. Numerical results and discussion

The results discussed from now on refer to a typical working day in winter and have been obtained considering 1000 scenarios. In particular, we have considered 20 different instances with the same number of scenarios to evaluate

the stability of the solutions. We note that the time required to solve every instance is still affordable and good in sample stability properties have been observed.

The proposed approach supports the retailer in the definition of dynamic tariffs and provides a daily operational plan in terms of amount of energy to purchase from the DAEM and the BM. The overall strategy is influenced by the safety level that, as other input parameters, can be dynamically modified to reflect changing market conditions.

Figure 4 shows the values of the retailer's objective function for different levels of $\gamma$. We recall that a safety margin equal to 0 is associated with a a risk-neutral position. In this case, the retailer aims to maximize the expected value and is not concerned about the reduction in profit that could occur in adverse situations. As expected, in this case the highest objective function value is achieved.

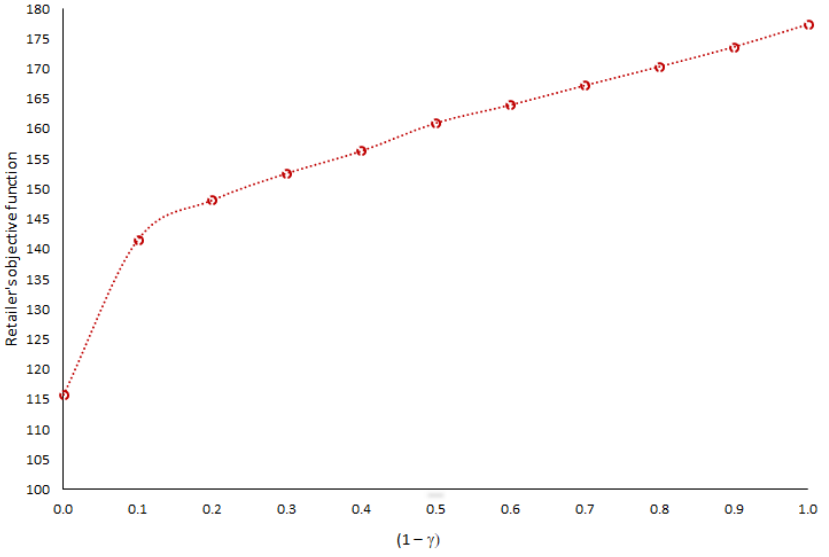



**Figure 4:** Retailer's objective value as function of the safety level

As the value of $\gamma$ increases, a decrease in the objective function value is observed. In such cases, the retailer would prefer "safer" solutions even if they are potentially less profitable. As an extreme case, the retailer could be interested in considering profit maximizing in the worst case scenario. When this criterion is selected, the minimum value for the objective function is achieved. By examining the results, we can appreciate the difference in objective function values associated with two extreme cases of neutrality and risk aversion. Specifically, in the case of risk neutrality, the value of the objective function is 66% higher than the corresponding value obtained for the risk aversion case.

Additional results are reported in Figure 5 where we compare the solutions obtained for the three different positions towards risk. In particular, we report the minimum, the maximum, and the quartile of the profit values associated with considered scenarios. Analyzing the results, we can observe that when a risk-neutral attitude is considered, a significant reduction in profit can be observed in the worst case scenario. For the test reported here, the reduction is around 42%, leading to a profit that is lower than that obtained when optimizing under the worst case. On the other hand, when implementing the strategy associated with the extreme risk-aversion position, lower profit values are obtained against those achieved when adopting the strategy proposed in the risk-neutral case. Looking at the figure, we can notice that the average profit achieved in the extreme case is lower than that associated with the moderate and neutral risk-aversion positions. In the risk-neutral case, the maximum value for the average profit is reached. Analyzing the results in more details, we observe that while for risk position there is a contraction of about 10% with respect to risk-neutral case, for a moderate risk-aversion level the average profit value is much closer with a reduction less than 4%. The results confirm that the neutral position strategy underestimates the unfavourable situations that may occur, whereas the extremely risk-averse case provides overly conservative solutions. Moderate risk-averse positions, calibrated by properly choosing the $\gamma$ values, may represent a good compromise.

Further experiments have been carried out to investigate the impact of the risk-aversion attitude on the retailer's pricing and supply decisions. Figure 6 shows the hourly offered tariffs. Looking at the results, we may observe that

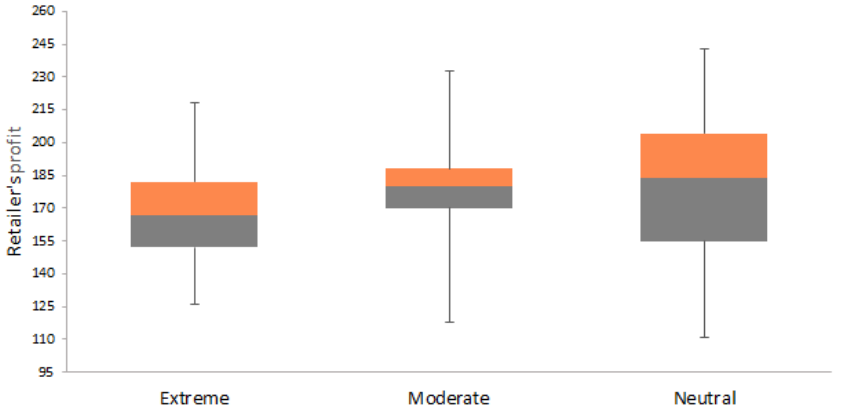



**Figure 5:** Solution analysis as a function of the risk-aversion level

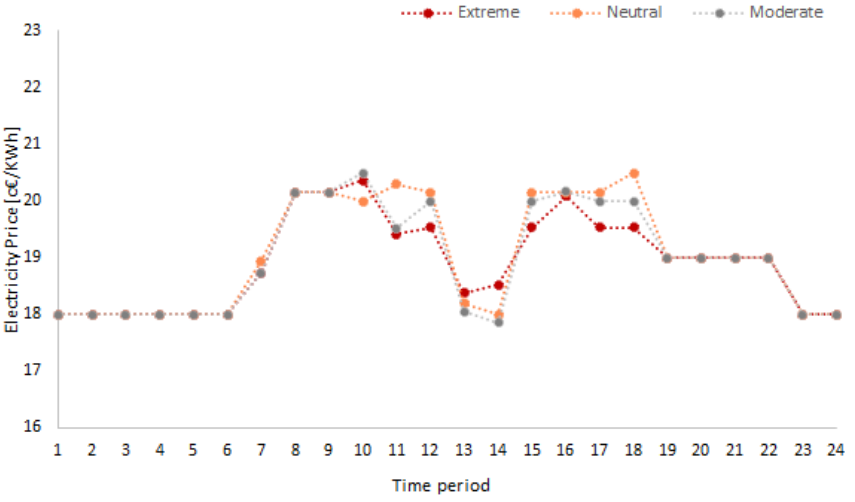



**Figure 6:** Offered rates for different risk-aversion positions

the choice of the safety level seems to have a limited impact on these decisions at least in the considered test instances. The optimal tariffs under different risk-aversion positions are similar, and only in the risk-neutral case, the offered rates are slightly higher for some time periods. Considering that, during the central hours of the day, the residual demand of the prosumager is typically limited because of the self production, the overall effect on the retailer's objective function is limited.

As for the supply policy, we report in Figure 7 the average cost distribution between the DAEM and the BM as function of the risk level. In the case of risk-neutrality, the supply strategy mainly relies on the DAEM market. This behavior could be explained by observing that, since the goal is to maximize the average profit, the retailer would prefer to buy larger quantities in the DAEM where prices are generally (at least in the experiments considered) lower than those of the BM. Nevertheless, the amount in excess to the demand could be resold if this situation would happen under some scenarios. For the other risk-averse positions, the share associated with the BM slightly increases, resorting to this market to guarantee the satisfaction of the prosumager request if under some realizations the electricity amount

purchased from the DAEM is insufficient.

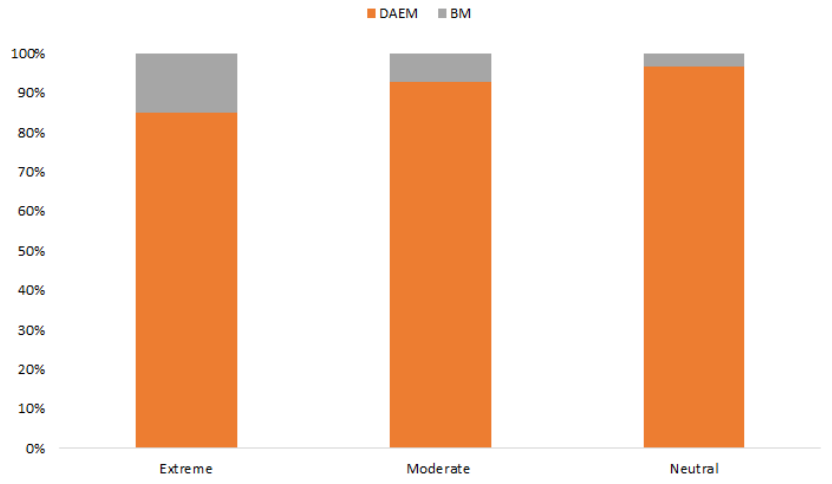



**Figure 7:** Retailer's supply policy under different risk attitudes

From now on, we present the detailed results obtained when considering a moderate risk averse position. We remind that once the tariffs are announced, the prosumager reacts to the retailer's decision by optimizing the management of the PV-BES energy system with the aim of minimizing the expected daily energy bill. The consumption pattern is scenario-dependent since a different management strategy might be adopted on the basis of the observed situation.

In the following, we detail the prosumager's reaction under a given scenario, noting that a similar behaviour has been observed for other realizations. Figure 8 shows how the prosumager's demand is covered throughout the day. We can observe that in period 3, when the rate is low enough, she buys an amount of electricity higher than the demand, which is stored into the battery and used during subsequent periods. The BES is also charged during the central hours of the day when the solar production is typically higher. Therefore, the residual demand can be covered via the BES discharge limiting the energy purchase from the retailer.

Figure 9 shows the management of the storage device, indicating the amount of energy charged, discharged, and the state of charge in each time period. The reported results confirm the economic advantage of the PV-BES solution nowadays widely adopted in the residential sector.

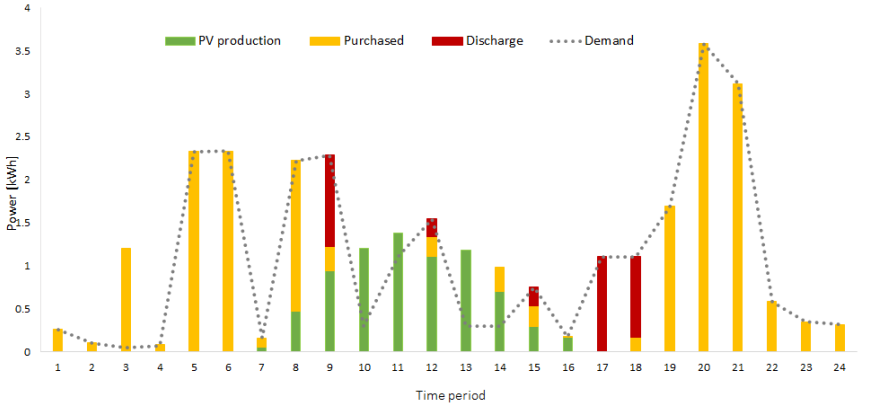



**Figure 8:** Prosumager's plan under a given scenario

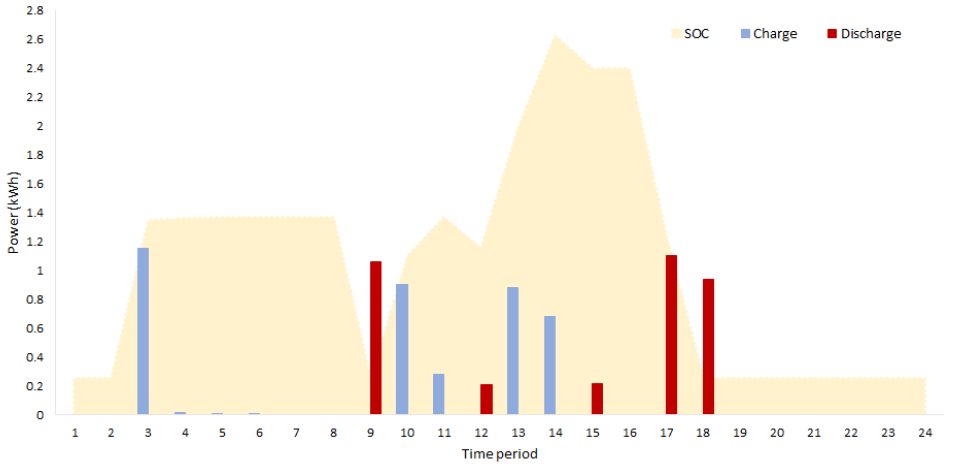



**Figure 9:** Storage management under a given scenario

## 5.4. The value of dealing with uncertainty and risk

It is well-known that stochastic programming is computationally more challenging than simply considering the deterministic counterparts of the problems. To highlight the importance of dealing with uncertainty and the adoption of a risk-averse approach, in what follows, we compare the proposed approach with two other alternatives: *i*) a deterministic model where the uncertain parameters are replaced by the corresponding expected values and *ii*) a stochastic risk-neutral approach.

### *5.4.1. Comparison with the deterministic approach*

As a metric of comparison with respect to the deterministic counterpart, we have used a variant of a well-known measure applied within stochastic programming, known as the Value of Stochastic Solution (VSS) (Ruszczyński and Shapiro, 2003). Such a measure, introduced for the traditional risk-neutral case, has been modified in Noyan (2012) to account for formulations incorporating a mean-risk objective function. In the proposed model, we do not have the same structure since we consider the CVaR as safety measure, thus we rename the corresponding metric as $\text{CVaR}_\gamma$-VSS. It is computed by the following formula:

$$\text{CVaR}_\gamma\text{-VSS} = \frac{\text{CVaR}_\gamma(\text{SP}) - \text{CVaR}_\gamma(\text{EVP})}{\text{CVaR}_\gamma(\text{EVP})}$$

Here $\text{CVaR}_\gamma(\text{SP})$ represents the objective function value of the proposed stochastic formulation, whereas $\text{CVaR}_\gamma(\text{EVP})$ indicates the value of $\text{CVaR}_\gamma$ corresponding to the optimal solution of the Expected Value Problem (EVP). Such a solution is obtained by solving the proposed stochastic BL model where the upper level first-stage variables are fixed to the optimal values corresponding to the deterministic (with expected values) BL problem. The higher the value of $\text{CVaR}_\gamma$-VSS, the larger the improvement in the solution quality gained by solving the stochastic model compared to its deterministic counterpart.
To evaluate the impact of the safety level on this measure, we show the values of the $\text{CVaR}_\gamma$-VSS for the risk-neutral, moderate, and extreme risk-averse cases in Figure 10.

Looking at the results, we may observe that when a risk-neutral position is considered, the gain deriving from the solution of the stochastic formulation with respect to the deterministic model is quite limited, with a value around to 2%. The advantage becomes more relevant as we increase the safety level, and, as expected, the highest value is achieved when considering the extreme risk-aversion position. In this case, the value of the measure is around 13%. We may notice that the difference between the computed values is higher when moving from an extreme risk-averse position to a moderate one with respect to what happens when passing from a moderate to a neutral position. This

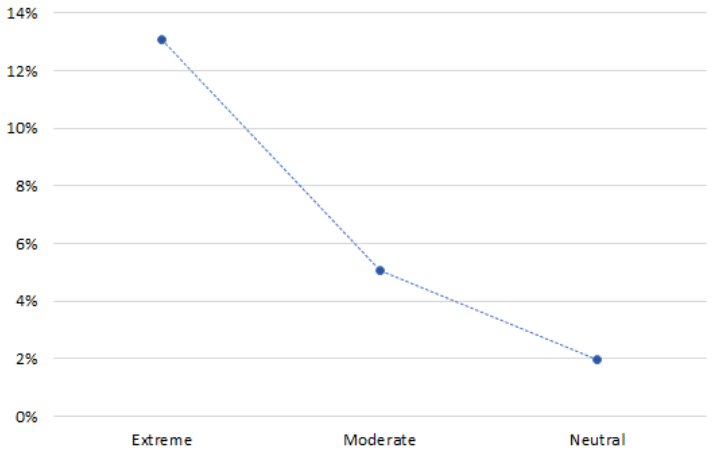



**Figure 10:** The $\text{CVaR}_\gamma$-VSS as a function of safety value

| | Safety level $\gamma$ | | | | | |
|---|---|---|---|---|---|---|
| | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.9999 |
| $\text{CVaR}_\gamma$- VRS | 1.0 | 1.1 | 1.3 | 1.6 | 2.4 | 11.1 |

**Table 3**
The $\text{CVaR}_\gamma$- VRS as a function of safety value

trend highlights the importance of dealing with uncertainty and risk in order to provide the decision maker with more reliable solutions.

#### *5.4.2. Comparison with the risk-neutral approach*

To further investigate the effect of the safety measure, we have compared the solutions of the proposed formulation with those obtained under a risk-neutral position. Similar to the VSS variant introduced above, we have computed another metric, refereed to as the Value of Risk Solution (VRS), which is defined as follows:

$$\text{CVaR}_\gamma\text{- VRS} = \frac{\text{CVaR}_\gamma(\text{SP}) - \text{CVaR}_\gamma(\text{RNP})}{\text{CVaR}_\gamma(\text{RNP})}$$

where $\text{CVaR}_\gamma(\text{SP})$ represents the objective function value of the proposed stochastic formulation and $\text{CVaR}_\gamma(\text{RNP})$ denotes the optimal objective value of the proposed stochastic BL model where the first-stage variables are set to the optimal values corresponding to the risk-neutral formulation (with the safety level of $\gamma = 0$). Table 3 reports the results obtained for increasing values of the safety level.

Looking at the results we may observe that the risk-neutral solution performs increasingly worse compared to the safety-based solutions. The difference is more evident for higher safety levels. In the case of extreme position the added value of using a risk aware formulation is quite significant reaching the value of 11 %. The results confirm the importance of considering randomness in the input parameters and integrate a safety measure to provide the retailer with a supporting tool able to cope with unfavorable situations that may occur.

### 5.5. Managerial insights

The extensive computational study carried out on a real test case and the analysis of the solutions allow to glean a number of practical insights that can support the retailer to deal with the electricity pricing problem.

The first managerial takeaways refer to the importance of explicitly dealing with the uncertainty and of integrating a safety measure. Even though the computational effort required to solve the proposed formulation is by far higher than that of the deterministic counterpart, the benefits of the stochastic formulation are evident. This is confirmed by the values the $\text{CVaR}_\gamma$- VSS and $\text{CVaR}_\gamma$- VRS computed for different values of the safety level.

More importantly, the added value is confirmed by the out-of-sample analysis carried out by using the real values of uncertain parameters. In particular, considering the nature and the dynamics of the problem, we have fixed the first-stage decisions, i.e. the electricity rates and the amount of electricity to purchase from the DAEM, deferring the second-stage decisions until uncertain data are revealed. Thus, on the basis of the realized values, the corrective actions can be evaluated and the real retailer's profit can be computed. Figure 11 reports the retailer's cumulative profit obtained by considering 10 consecutive working days in winter and associated with the three positions of extreme, moderate and neutral risk-aversion. The basis of comparison is represented by the cumulative profit, obtained when implementing the recommendations provided by the solution of the deterministic model.

Looking at the figure, we notice that the difference among the profit values corresponding to the three safety levels is

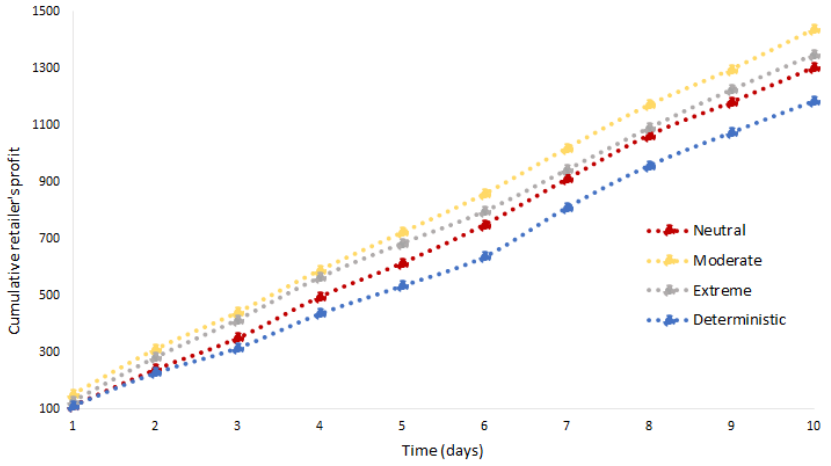



**Figure 11:** Out-of-sample analysis: Retailer's cumulative profit

quite small, with better performance achieved in the case of moderate risk-aversion. On the contrary, the advantage of the stochastic solutions over the deterministic counterpart is more evident and higher benefits can be gained in the long run. Overall, the integration of the safety measure into the stochastic formulation enables the retailer to control the variability of the profit based on the adopted safety level and provides him/her with a useful tool for decision-making under uncertainty.

Another insight refers to the use of a rolling strategy in the design of dynamic pricing schemes. The proposed uncertainty-aware approach is solved every day using scenarios generated considering updated and thus more reliable information. This adaptive structure exploits the potential offered by big data in providing more reliable forecasts.

We also underline that the proposed model has a general structure that allows to accommodate also other time-differentiated pricing options. One of the most used options is, for example, the ToU. We have run our experiments also considering this alternative scheme. According to the organization of the Italian market taken as reference, three main blocks (for working days), peak (8 a.m.-6 p.m.), intermediate (7 a-m., 7-10 p.m.) and off-peak (11-12 p.m., 1-6 a.m) have been considered. The results show that for the typical day investigated in the experiments a slight higher profit has been registered in the case of the real time option. The slight increase, about 3% (average value on the three safety levels), can be explained observing that in this case the retailer could better exploit the dynamics of the wholesale electricity prices.

For the prosumager, acting as a follower, the empirical evidence demonstrates the importance of the optimal management of the local energy system by the solution of a stochastic model. The results confirm that the integration of the battery with the PV panels allows to mitigate the effects of the intermittent nature of the solar production and to achieve significant economic advantage.

| Symbol | Value |
|---|---|
| $C$ | 3.8 |
| $\mu_1$ | 0.05 |
| $\mu_2$ | 0.95 |
| $\tau_c$ | 0.95 |
| $\tau_d$ | 0.95 |
| $\eta_c$ | 0.98 |
| $\eta_d$ | 0.98 |
| $soc_0$ | 0.19 |

**Table 4**
Input parameters: storage system

## 6. Conclusions and future research directions

This paper presents a stochastic BL approach for the electricity tariff design under uncertainty. The interaction between retailer (leader) and prosumager (follower) is modeled as a Stackelberg game. Both players solve an optimization problem subject to uncertainty related to varying market prices, weather-related variables and prosumager's load. To account for the retailer's attitude towards risk, the upper level problem includes a safety measure to maximize. The model allows to determine the dynamic pricing rates delivering the average profit that can be gained in a given percentage of unfavorable realizations corresponding to the uncertain parameters and the optimal load pattern that minimizes the expected prosumager's electricity bill.

The stochastic BL problem is reformulated as a single level one and different approaches to deal with the lower level problem and the non linearity in the objective function are analyzed and empirically investigated. A large number of numerical experiments have been carried out on real test cases. The results have shown the efficacy of the proposed approach as a tool to define pricing schemes that reflect the retailer's risk attitude underlining the importance of explicitly dealing with uncertainty in the main parameters involved in the decision process and to integrate a safety measure.

Different issues are left for future investigation. Extending this study for smart prosumager with flexible loads, i.e. loads that can be postponed provided that they are operated in a given time window, is an interesting research for future work. In this case, the lower level problem would contain binary decision variables related to the flexible loads. This requires the design of a tailored solution method since in this case a single level reformulation can not be provided. Another extension would consider the replacement of the upper level actor with an aggregator instead of a retailer. Aggregator represents an emergent figure in the energy sector and can be seen as an entity that manages a coalition of end-users, typically smart prosumagers. The aggregator typically has access to renewable energy sources and/or traditional energy plant in order to guarantee a minimum level of electricity production. In this case the corresponding upper level problem would be more involved including additional variables (such as binary) and constraints.

Finally, since the aggregator is typically called to manage different type of end-users (i.e. residential, public, and private consumers) an extension of the model to a multi-follower setting could be another research avenue together with the design of efficient solution approaches.

## Appendix

Table 4 reports the technical details of the prosumager's BES system. Figure 12 shows the average consumption pattern for a winter day (see also Violi, Beraldi and Carrozzino (2022)), whereas Figure 13 reports the average production from PV panel during the hours of four typical days, each representative of a different season.

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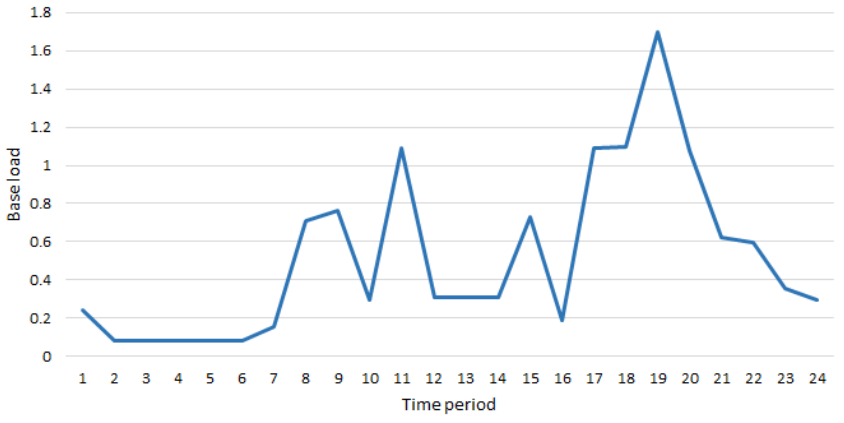



**Figure 12:** Base load demand for weekdays in winter

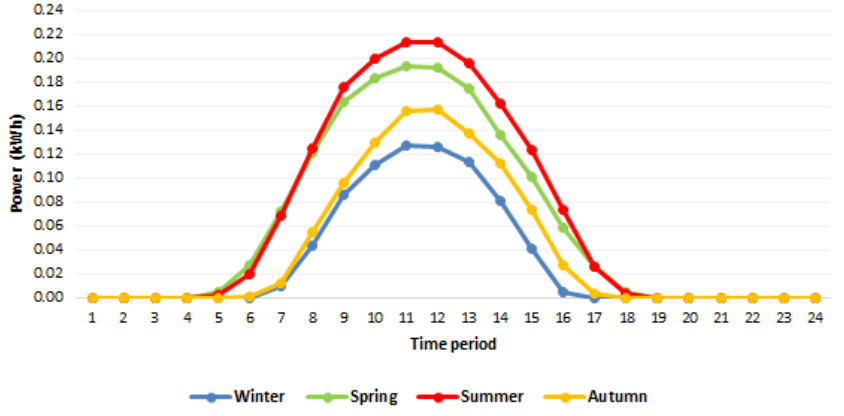



**Figure 13:** The PV energy production for typical days

## References

Algieri, A., Beraldi, P., Pagnotta, G., Spadafora, I., 2021. The optimal design, synthesis and operation of polygeneration energy systems: Balancing life cycle environmental and economic priorities. Energy Conversion and Management 243, 114354.

Anjos, M.F., Brotcorne, L., Gomez-Herrera, J.A., 2021. Optimal setting of time-and-level-of-use prices for an electricity supplier. Energy 225, 120517.

Antunes, C.H., Alves, M.J., Ecer, B., 2020. Bilevel optimization to deal with demand response in power grids: models, methods and challenges. TOP 28, 814–842.

Askeland, M., Burandt, T., Gabriel, S., 2020. A stochastic mpec approach for grid tariff design with demand-side flexibility. Energy Systems , 1–23.

Aven, T., 2016. Risk assessment and risk management: Review of recent advances on their foundation. European Journal of Operational Research 253, 1–13.

Beraldi, P., Conforti, D., Triki, C., Violi, A., 2004. Constrained auction clearing in the italian electricity market. 4OR 2, 35–51.

Beraldi, P., Khodaparasti, S., 2022. A bi-level model for the design of dynamic electricity tariffs with demand-side flexibility. Soft Computing .

Beraldi, P., Violi, A., Scordino, N., Sorrentino, N., 2011. Short-term electricity procurement: A rolling horizon stochastic programming approach. Applied Mathematical Modelling 35, 3980–3990.

Brotcorne, L., Lepaul, S., von Niederhäusern, L., 2021. A rolling horizon approach for a bilevel stochastic pricing problem for demand-side management. arXiv:2102.13634.

Burtscheidt, J., Claus, M., Dempe, S., 2020. Risk-averse models in bilevel stochastic linear programming. SIAM Journal on Optimization 30, 377–406.

Bussieck, M.R., Meeraus, A., 2007. Algebraic Modeling for IP and MIP (GAMS), in: Annals of Operations Research. Kluwer Academic Publishers-Plenum Publishers. volume 149(1). special edition History of Integer Programming: Distinguished Personal Notes and Reminiscences, Guest Editors: Kurt Spielberg and Monique Guignard-Spielberg, pp. 49–56.

Colson, B., Marcotte, P., Savard, G., 2007. An overview of bilevel optimization. Annals of Operations Research 153, 235–256.
Costa, A., Ng, T.S., Foo, L.X., 2017. Complete mixed integer linear programming formulations for modularity density based clustering. Discrete Optimization 25, 141–158.
Dadashi, M., Haghifam, S., Zare, K., Haghifam, M.R., Abapour, M., 2020. Short-term scheduling of electricity retailers in the presence of demand response aggregators: A two-stage stochastic bi-level programming approach. Energy 205, 117926.
Di Somma, M., Graditi, G., Heydarian-Forushani, E., Shafie-khah, M., Siano, P., 2018. Stochastic optimal scheduling of distributed energy resources with renewables considering economic and environmental aspects. Renewable Energy 116, 272–287.
Do Prado, J.C., Qiao, W., 2020. A stochastic bilevel model for an electricity retailer in a liberalized distributed renewable energy market. IEEE Transactions on Sustainable Energy 11, 2803–2812.
Dutta, G., Mitra, K., 2017. A literature review on dynamic pricing of electricity. Journal of the Operational Research Society 68, 1131–1145.
Escudero, L.F., Monge, J.F., Rodríguez-Chía, A.M., 2020. On pricing-based equilibrium for network expansion planning. A multi-period bilevel approach under uncertainty. European Journal of Operational Research 287, 262–279.
Ferrara, M., Violi, A., Beraldi, P., Carrozzino, G., Ciano, T., 2021. An integrated decision approach for energy procurement and tariff definition for prosumers aggregations. Energy Economics 97.
Garifi, K., Baker, K., Christensen, D., Touri, B., 2018. Control of energy storage in home energy management systems: Non-simultaneous charging and discharging guarantees. arXiv preprint arXiv:1805.00100 .
Goshu, N.N., Kassa, S.M., 2020. A systematic sampling evolutionary (sse) method for stochastic bilevel programming problems. Computers & Operations Research 120, 104942.
Grimm, V., Orlinskaya, G., Schewe, L., Schmidt, M., Zöttl, G., 2020. Optimal design of retailer-prosumer electricity tariffs using bilevel optimization. Omega , 102327.
Heitsch, H., Römisch, W., 2003. Scenario reduction algorithms in stochastic programming. Computational optimization and applications 24, 187–206.
Kleinert, T., Labbé, M., Plein, F., Schmidt, M., 2020. Technical note-there's no free lunch: On the hardness of choosing a correct big-m in bilevel optimization. Operations Research 68, 1716–1721.
Mitra, S., Domenica, N.D., 2010. A review of scenario generation methods. International Journal of Computing Science and Mathematics 3, 226–244.
Noyan, N., 2012. Risk-averse two-stage stochastic programming with an application to disaster management. Computers & Operations Research 39, 541–559.
Rockafellar, R.T., Uryasev, S., 2000. Optimization of conditional value-at-risk. Journal of Risk 2, 21–41.
Rockafellar, R.T., Uryasev, S., 2002. Conditional value-at-risk for general loss distributions. Journal of Banking & Finance 26, 1443–1471.
Ruszczyński, A., Shapiro, A., 2003. Stochastic Programming, Handbook in Operations Research and Management Science. Elsevier Science, Amsterdam.
Sekizaki, S., Nishizaki, I., et al., 2019. Decision making of electricity retailer with multiple channels of purchase based on fractile criterion with rational responses of consumers. International Journal of Electrical Power & Energy Systems 105, 877–893.
Setak, M., Feizizadeh, F., Tikani, H., Ardakani, E.S., 2019. A bi-level stochastic optimization model for reliable supply chain in competitive environments: Hybridizing exact method and genetic algorithm. Applied Mathematical Modelling 75, 310–332.
Soares, I., Alves, M.J., Antunes, C.H., 2020a. Designing time-of-use tariffs in electricity retail markets using a bi-level model–estimating bounds when the lower level problem cannot be exactly solved. Omega 93, 102027.
Soares, I., Alves, M.J., Antunes, C.H., 2020b. A deterministic bounding procedure for the global optimization of a bi-level mixed-integer problem. European Journal of Operational Research .
Violi, A., Beraldi, P., Carrozzino, G., 2022. Dealing with the stochastic prosumager problem with controllable loads. Soft Computing .
Zare, M.H., Borrero, J.S., Zeng, B., Prokopyev, O.A., 2019. A note on linearized reformulations for a class of bilevel linear integer problems. Annals of Operations Research 272, 99–117.
Zugno, M., Morales, J.M., Pinson, P., Madsen, H., 2013. A bilevel model for electricity retailers' participation in a demand response market environment. Energy Economics 36, 182–197.