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Damping in Composite Materials: Properties and Models

A. Treviso[a,b,*], B. Van Genechten[a], D. Mundo[b], M. Tournour[a]

[a] *Siemens Industry Software N.V., Leuven, Belgium*
[b] *Università della Calabria, Dipartimento di Ingegneria Meccanica, Energetica e Gestionale, Rende, Italia*

Abstract

The present review aims at gathering the available literature on damping in composite materials. A chronological review of test methods for damping estimation is presented in order to describe the time-line of the theoretical knowledge development in this field. In the last years many new material configurations have emerged that deserve investigation, such as nano-composites, hybrid laminates and sandwich materials. Damping models specifically meant for non-homogeneous materials are reported to provide a background for understanding this problem. Although not widely exploited yet, fibre reinforced polymers has the potential to be tailored for damping by acting on constituents, geometry and boundary conditions. Nano-composites, for instance, are shown to possess a high potential for damping purposes. New hybrid and sandwich-type structures are emerging as noise and vibration control solutions in lightweight applications. The effort devoted to mathematical and numerical model in view of Finite Element integration of damping properties is also addressed. Finally, the conclusions summarise the ideas of the authors on needed steps to advance the state-of-the-art in each of the described topic.

Keywords: A. Polymer-matrix composites (PMCs); B. Internal friction/damping; C. Analytical modelling; D. Mechanical testing

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[*] Corresponding author
Email address: `alessandra.treviso@siemens.com`, `alessandra.treviso@unical.it` (A. Treviso)

1. Introduction

The use of composite materials has constantly increased during the last decades. Several examples show that composite materials have entered the industry as a viable alternative to traditional materials. Aerospace and defence industries have been pioneers in this field but the attractive performance properties of composite materials have soon triggered their application for the production of sport equipment, boats, reinforcement components in civil engineering and energy applications. In the transport industry, aircraft design has taken the lead and the most recent aircrafts are made of composite for up to 60% of their weight. Automotive industry has been using composites for luxury and competition cars for several years and it is now employing them also in the series production of passenger vehicles.

Since their first appearance and applications, most researchers have focused on static characterization and effective modelling of inhomogeneity and anisotropy and an extensive literature is available on this topic.

The use of composites for both primary and secondary structural components calls for a deeper understanding not only of their static characteristics but also of dynamic ones. In general, the use of static stiffness for the prediction of natural frequencies is acceptable. Damping properties, however, are difficult to assess and therefore difficult to account for in the design and analysis stage. To the knowledge of the authors, few reviews [1–3] have been published on this topic in the past with a more limited scope. Nonetheless, the damping of composite materials can be several orders of magnitude higher than that of traditional engineering materials, making them appealing also for components undergoing dynamic loading. The exploitation of material damping could also improve the general performance of composite structures compared to traditional vibration absorbing treatments. In fact, the weight of the treatments and the weight of the structures are in some cases comparable such that adding them to a base panel has a significant effect on the natural frequencies and mode shapes of the structure.

This review aims at gathering all the knowledge that has been gained in this field over the last years, treating both theoretical formulations and experimental procedures developed. The first part reviews the experimental techniques that have been developed to estimate damping in composite structures, followed by a description of the parameters affecting damping with the support of quantitative examples. The last section of part two covers the emerging hybrid composites and nano-composites. Finally, the last paragraph covers the approaches currently available to model damping, with particular focus to those typically employed for composite materials.

2. Experimental characterisation

Experimental measurements are essential in the study of damping of composite materials either for validation purposes or for material characterisation. However, proper techniques are needed to tackle the low weight and high stiffness. For example, standard *Dynamical Mechanical Analysis* (DMA) testing is most of the times not possible because of the high modulus [4]. Creep or relaxation tests can be used to determine linear viscoelastic models' parameters but they are usually time-consuming and require strict temperature and loading control [5]. A chronological review of test methods offers the reader the chance to follow the development of the knowledge in the field of damping. Since the beginning, the main concerns were related to the mass effect of traditional accelerometers and to supporting or clamping conditions. In reference [6], Sola and Jetté failed to validate their model because of non-perfect clamping conditions and accelerometers mass, both of which were not included in the model; Talbot and Woodhouse [7] found significant discrepancies between predicted and experimental values. The effect of different constraints on the modal damping is extensively studied by Maheri [8] on square plates. The way constraints act on damping is related to the variation in stiffness distribution. Such effect is shown also in Li and Narita [9]. Asymmetric constraining conditions may induce vibration coupling between normal and transverse stress components. The effect of coupling on damping has been studied by Hwang et al. [10, 11] who quantified the contribution of coupling in terms of strain energy and found the optimal ply angle that maximizes it. Zabaras and Pervez [12] accounted for the coupling between bending and shear in the formulation of the laminate stiffness matrix.

Most of the tests that will be described in the upcoming section deals with unidirectional plies. As thoroughly explained in [13], the damping properties of orthotropic plies, which constitute the fundamental unit of laminates, is characterised by three damping coefficients along the material directions and on the plane they define. Such coefficients can be determined by analysing beams' vibrations and the frequency-dependent characteristic of the loss factor determined by varying the length of the beam.

In 1972 Wright [14] tested glass and carbon reinforced polyester beams by suspending them at the fundamental mode node thus reproducing free-free conditions. Beams were excited by a sinusoidal force and displacements were measured by an optical contactless probe. The damping was then measured by recording the oscillation decay. Adams and Bacon [15] stressed the importance of using free-free vibration conditions to avoid extraneous damping mechanisms; moreover, to exclude the effect of amplitude-dependence steady state conditions are employed. These conditions are met by exciting a CFRP beam electromagnetically by mounting a coil at its midpoint. The mode amplitude of the specimen is measured to obtain the energy stored and calculate the SDC. Due to the low relative weight difference between the coil and the specimen, the mass effect has to be taken into account. In a successive paper [16] they addressed the issue of air damping, that can become a relevant source of dissipation for big displacements of the beam, by suggesting *in vacuo* tests for low damped material such as CFRP. Possible sources of discrepancies between predicted and measured values are also proposed. Guild and Adams [17] recognised in the apparatus used by Adams and Bacon a source of uncertainty due to the clamping pressure of the coil on the beam that may introduce cracks which increase the damping capacity. They propose two new apparatuses, one for free-free vibration and one for cantilever vibration. Electromagnetic excitation of two coils in free-free tests and one coil in cantilever tests is used but the coil clamps are improved to be stiffer in order to exclude spurious contributions to damping. Using the same approach of Adams and Bacon, the SDC is calculated and similar trend are found with both methods though different values are obtained due to the different boundary conditions and possibly a different mass effect. In reference [18], Lin et al. tested supported glass and carbon reinforced epoxy plates. To limit the contribution of boundary conditions, soft rubber foam supports are placed under the expected position of the modes' nodes. Steady-state vibration and hammer impacts are used to estimate the SDC of the plates and non-contact transducers are employed to record the response. Suarez et al. [19] described random and impulse techniques to measure the viscous damping ratio with the half-power bandwidth method. The first makes use of an electromagnetic shaker and an eddy current probe, measuring the damping by means of the half-power bandwidth method. The impulse technique relies on an electromagnetic hammer to ensure a higher level of reproducibility. The main advantages of the first setup are: good control of the force level and no leakage, but the fixing and excitation conditions must be accurately controlled. The impulse method is more suitable for *in situ* testing, especially for health-monitoring purposes, being damping one of the most sensitive parameter to damage due to additional frictional dissipation occurring at cracks and delamination sites [20, 21]. In reference [22], the apparatus is improved to allow extensional damping measures. Crane and Gillespie [23] developed a similar impulse technique apparatus, underlining the importance of avoiding standard accelerometers to reduce the contribution coming from the non-negligible mass addition, with respect to the specimen mass, and from the cables. The clamping system is enriched by guiding rails to avoid eccentric loading and possible uncontrolled vibration couplings. Maheri and Adams [24] used several contacting and non-contacting devices for both the excitation and the sensing to conclude that the laser sensor gives the most accurate displacement measures, which are then used to estimate the SDC. Gibson [25] reviewed different modal vibration techniques for the static and dynamic properties of composites at both specimen and component level. Vibration damping is proposed by Kyriazoglou [26] in the framework of a hybrid simulation methodology. In vibration damping beams are vibrated in free-free flexure in their fundamental mode; the vibration is driven by magnets attached at both ends; the excitation is a sinusoidal signal; tests are run in vacuo and displacement are measured by a laser vibrometer; finally, SDC is calculated. Vasques et al. [27] described direct and indirect method used for damping evaluation and proposed a novel test-rig for the characterization of damped materials. All the above mentioned methods are based on vibration testing which, according to Stevensons [28], prevent from distinguishing between material and structural damping.

Though focused on homogeneous materials, the paper by Vanwalleghem et al. [29] gives a detailed description of all the external factors possibly contributing to damping and provides guidelines on how to limit their effect.

3. Damping phenomena

Compared to metals, composite materials show generally a higher damping capacity. The main reason is the viscoelasticity of the polymeric matrix. As for stiffness and strength, also damping can be tuned by properly choosing composites constitutive parameters such as fibre aspect ratio, stacking sequence and constituents properties. However, in most cases optimal results for damping properties lead to insufficient performance in terms of strength and stiffness and a trade-off has to be found. The topic of optimization of strength and damping of composite structures is beyond the scope of this review, which is limited to the description of the effect that structural and operational parameters have on the dissipative behaviour of composite materials.

Most studies in this area have been conducted on polymer composites since these are the most appealing due to ease of manufacturing, low cost and high dissipation characteristics.

Additionally, nano-composites, hybrid composites and sandwiches are worth to mention since they are becoming alternative solutions for weight reduction in many fields.

3.1. Short and long fibre reinforced polymers

Constituent materials

The primary source of damping in composites is the inherent damping of their constituents. The viscoelasticity of the polymeric matrix is hence the main contributor. Thermoplastic polymers show high dissipation compared to thermosets though the latter are generally preferred for their higher stiffness and better adhesion properties [30]. By increasing the matrix volume fraction the damping will increase at the expenses of stiffness and strength. Ni and Adams [31] showed the trend for carbon fibre reinforced polymer (CFRP) and glass fibre reinforced polymers (GFRP) damping which increases parabolically as the matrix volume fraction (Vf) increases, reaching an almost constant value for Vf=0.6. Lin et al. [18], Hadi and Ashton [32], Haddad and Feng [33] confirmed this tendency. Among thermosets, Crane and Gillespie [23] found that epoxy has better dissipation characteristics than PEEK, despite the bi-phasic nature of the latter. The reinforcement contribution is the result of excitation, interaction with the matrix and bulk damping properties. When fibres are loaded along their axis they drive the response of the laminate also in terms of damping as reported by Vantomme [34] and Berthelot and Sefrani [35] who accounted for the motion of the fibres in a viscoelastic matrix in both the longitudinal and transverse direction as a source of dissipation. Comparison of experimental results also showed a predominance of fibres-matrix interaction phenomena in the longitudinal direction. From their experimental campaign on plates Maheri and Adams [36] concluded that the *Specific Damping Capacity* (SDC) increases when the bending axis is parallel to the axis of the fibres, being this the loading condition that causes the highest shearing of the matrix. The interaction between matrix and fibres plays a crucial role in load transmission and eventually in the resistance of the composite. To ensure that debonding does not occur, fibres are treated with bonding agents which contribute to damping. Nelson and Hancock [37] studied the interfacial slip in short fibre composites. They found that the weaker the bond between fibres and matrix the higher the hysteresis due to the relative motion of fibres. This is in accordance with results reviewed by Doebling et al. [38] about the increase of damping in presence of damages, such as debonding and delaminations. Vazquez et al. [39] coated glass fibres with mono- and di-epoxy resin and studied the damping as a function of the coating treatment concluding that increasing thickness can lead to a decrease in damping due to decreased load transmission. Finegan and Gibson [40] considered PVC coated copper fibres and documented an increase of loss factor with PVC volume fraction increase however at the expense of stiffness properties. Remillat [41] gave a general description of the problem of composites with an intermediate phase between fibres and matrix. Chaturvedi and Tzeng [42], Hwang and Gibson [43], Vantomme [34] and Chandra et al. [44] considered the interphase as the result of the production process and accounted for it in the micro-mechanical modelling of short and long fibre composites. However, no direct information is available on the properties of the interphase and intermediate properties between matrix and fibres are usually assumed. In all these studies the fibres volume fraction was kept constant and as a result, the loss factor of the three-phase system decreased in response to a reduction in matrix volume fraction. Finally, when comparing different fibre materials Berthelot and Sefrani [45] found that Kevlar gives the highest damping compared to glass and carbon; glass and carbon were compared by Crane and Gillespie [46] and Wright [14] and they all found that glass fibres have better damping properties.

Orientation and stacking sequence It is not surprising that ply orientation and layup are the most investigated factors since they are fundamental in defining global composite properties. The research in this field is aimed at finding a) the ply angle that ensures the highest damping and b) its location in the stack up. As for experimental characterisation, the analysis stems from the orthotropic plies' damping coefficients in the longitudinal, transverse and in-plane shear material directions. Data on the optimal angle are not always in agreement but all authors agree on looking for configurations that maximize the shear stress components. A milestone study is the one by Adams and Bacon [47], in which unidirectional, angle-ply, cross-ply and general cases are tested and modelled . For the unidirectional samples the damping is found to have a maximum value at around 35 degrees in flexure and at around 45 degree in torsion. For angle-ply the peak appear between 40 and 50 degrees in both bending and torsion and for cross-ply the most important factor is the cross-ply ratio that defines the relative number of 0 and 90 degrees plies. These trends are confirmed in later papers by Adams and Maheri [48, 49], Berthelot et al. [13] and El Mahi et al. [50]. Wray et al. [51] performed an analogous analysis on short GFRP finding the optimal angle that proved to vary for different volume fractions. Yim [52] provided variation of loss factor with fibre orientation comparing Ni and Adams theory to his model that takes into account also the loss factor associated with the Poisson's ratio. Once the optimal angle is found the stacking sequence is analysed. All authors agree in placing the ply that is most sensitive to shearing in the location of maximum shearing within the laminate. The layup affects also the magnitude of interlaminar stresses: due to the stiffness mismatch at the interface of plies with different orientation, in-plane and out-of-plane shear stresses arise. Hwang and Gibson [53, 54] analysed their contribution over a wide range of laminate angles identifying in the maximization of the shearing components a way to maximise damping as well. The effect of layup is extensively treated by Maheri in [8] where he focuses on square symmetric plates in order to exclude the aspect ratio and coupling effects respectively. For laminates that do not experience a steep change in stiffness because of a relatively uniform distribution fibres' lay with respect to direction, damping is also relatively uniformly spread. This effect can be appreciated by observing the variation in modal under different boundary conditions. The variation experienced by cross-ply layups is around four times higher than that experienced by an angle-ply layup, with $\theta = 45°$. Finally, Ohta et al. [55] analysed the effect of stacking sequence observing the different convergence achieved by different plate theories as the layup is changed.

Geometrical properties

Properties of composite materials depend on reinforcement geometrical properties. This is especially true for short fibres reinforced polymer whose damping characteristics are affected by the length and the shape of the fibres. The aspect ratio determines the transverse shear stress distribution along the fibres and therefore the amount of shearing that the matrix will undergo. Gibson et al. [56] found an optimal fibre aspect ratio for maximum damping which is however below the minimum one required for maximum stiffness. Furthermore, it appears that whiskers allow to reach higher damping values. Similar trends were found by Sun et al. [57, 58], Suarez et al. [59] and Subramanian et al. [60] who identified in the fibre ends the zone of maximum energy dissipation due to the weakness of the bonds combined with a high stress concentration. The fibres packing has been studied by Tsai and Chi [61] who compared square edge, square diagonal and hexagonal packing configuration to conclude that square diagonal packing provided the best damping in both free-free and cantilever configurations. In reference [62], Chandra et al. reviewed different modelling approaches to asses the influence of fibre geometry. The geometry of the components has also been investigated in terms of beam aspect ratio by Yim and Gillespie [63] and in terms of thickness by Crane and Gillespie [46]. When long or short fibres are replaced by fabrics different damping values should be expected. Guan and Gibson [64] calculated the loss factor of woven materials according to the mosaic model, Mishra and Sahu [65] experimentally studied the damping in woven reinforced plates at varying boundary conditions and aspect ratio and Lei et al. [66] compared the dynamic properties of five different woven structures addressing the friction between the yarns as a discriminating factor when fibre volume fraction is fixed.

Temperature and frequency effects

Finally, temperature and frequency modify the behaviour of the matrix and affect damping. Benchekchou et al. [1] analysed resin blends in order to obtain a range of glass transition temperatures, at which damping has generally a peak, for a CFRP. Generally speaking, temperature softens the matrix causing an increase in loss factors [67] but with detrimental effects on stiffness. Frequency dependence of polymer composites is a controversial subject. In most applications, the frequency range is limited and viscoelastic properties of the matrix can be considered as constant. Hadi and Ashton [32] found that for different ply angles and volume fractions the SDC was independent of frequency in the range 0-1000 Hz. In the same range

Crane and Gillespie [46] found different relation at 0 and 90 degree for glass and graphite composites. In reference [68] linear trends are also found though the slope is low.

3.2. Nano-composites

Many review papers are available in the field of nano-composites with particular focus on the use of nano-tubes [69–72]. For the sake of completeness we will review the main characteristics of this class of composites addressing the interested reader to the cited papers.

Nano-composites employ nano-scale materials as reinforcements. Depending on the application, carbon nano-tubes, ceramic or polymeric particles can be used. Among these, carbon nano-tubes are the most attractive because they can provide both strength and damping to the hosting matrix. The main dissipation mechanism is frictional sliding [4], that is higher in nano-composites due to the larger interfacial area they offer compared to traditional reinforcements. An extensive description of the *stick-slip behaviour*, responsible for the damping, can be found in reference [73] where more results relating the variation of the loss factor to the volume fraction of the nano-tubes are also given.

De Borbón et al. [74] compared different nano-tube types in both a sandwich beam and an epoxy beam. The addition of nano-tubes caused an increase in both cases; depending on the type and configuration different increments could be achieved. The use of a nano-tube film is reported by Koratkar in [75], where a 200% increase in structural damping is obtained by adding the nano-tube film reinforcement to a sandwich structure. In a more recent paper Koratkar at al. [76] investigated the use of single-walled nano-tubes proving an even higher increase in damping due to the frictional sliding of nano-tubes in the matrix, which in turns depends on the quality of their dispersion and separation.

Finally, an example of how the damping varies with respect to the nano-tube volume fraction is given by Montazeri and Montazeri in [77], also with regard to the physics of the problem.

3.3. Sandwich-type composites, constrained and unconstrained viscoelastic layers

Sandwich structures are obtained by interlaying a thick, lightweight material between two thin, stiff skins. Although belonging to a particular class of composite materials, many configurations are available. These may include classic laminates or metal foils as skins and honeycomb or soft polymers as core. For damping purposes the use of a soft, viscoelastic material is preferred for the core while both metals and laminates can be used for the skins. The main damping mechanism in viscoelastic-core sandwiches is related to the high transverse shear stresses that develop at the skin-core interface due to the high stiffness mismatch. The effectiveness of viscoelastic layers in increasing structures' loss factor has been proved by many authors [78–82].

Ungar [83], Kerwin [84], Mead [85], Mead and Markus [86] were among the first who paid attention to the issue of damped vibration of sandwich structures. In [84] Kerwin gives the damping as a function of frequency, layer thickness and temperature. In [85, 86] Mead and Mead and Markus found that the damped modes were real only at a particular frequency at which the imaginary part changed sign. The implication is that by changing the boundary conditions the loss factor vs. frequency curves must change. As a matter of fact, boundary conditions are now commonly recognised as a sensitive parameter and their effect is especially considered in the experimental evaluation of damping properties, as will be discussed in a later section. In two accompanying papers Berthelot and Berthelot and Sefrani [87, 88] gave analytical formulations for the damping of single and double interleaved viscoelastic layers and experimental results for the same configurations. Fixing the orientation of the laminate skins, the loss factor in the presence of the viscoelastic layers increases by an order of magnitude. The loss factor was shown to increase with increasing thickness of the viscoelastic layer and with decreasing length of the test specimens. When the two configurations are compared it appears that a single but thicker layer is more efficient than two layers, even if the total viscoelastic fraction is the same. This behaviour depends on the location of maximum shear strains and associated energy, i.e. the middle plane. In reference [89], the variation of loss factor with respect to core thickness is found to be linear, independently of the skin materials while its absolute value is a function of the skins' stiffness and in particular of the difference between the core and the skins stiffness. In reference [90], the effect of temperature is investigated showing that the highest damping is obtained when the viscoelastic core operates near its glass transition temperature. Similar results are found by Rao et al. [91] and by Chen and Levy [92] where temperature gradients were caused by the presence of a SMA layer for active control of vibration. In reference [93], the effect of the beam aspect ratio and of the skin-to-core thickness is observed. In the first case, there exist an aspect ratio for which the loss factor is maximum; in the second case an increase in core thickness causes an increase in damping especially in the region of low aspect ratios. Introducing a viscoelastic layer over the entire surface of the beam may not be the optimal configuration both for damping and weight reasons. Fotsing at al. [94] studied a honeycomb beam with laminate skins and interleaved viscoelastic layers. The viscoelastic layer was either distributed over the entire surface of the beam or applied in patches of different size at different locations. Experimental tests proved that the patches were more efficient in damping the vibrations and that the optimal location was around the beam nodes, being points of maximum shear strains. The effect of the type of constraint is also investigated by comparing closed edge treatments and open edge treatments. The first appears to be more efficient because of the higher transverse shear stress gradients undergone by the viscoelastic layer. However, the extension of the closed edge has to be tuned in order to allow for the shear deformation of the viscoelastic layer. An alternative approach is proposed by Meaud et al. [95] who built a layered composite by alternating a lossy viscoelastic polymer and steel.

With regard to sandwiches with composite skins, it is necessary to point out that interleaving the viscoelastic layer is not a trivial task. Co-curing is necessary, creating an issue in manufacturability because of the different thermal behaviour of the laminate matrix and the viscoelastic layer [1, 96]. This approach however would allow weight savings and more damping compared to the application of the viscoelastic layer on top of the structure, constrained by an additional foil of material [3].

The application of unconstrained viscoelastic layers on top of structure solves the issue of co-curing and still provides additional damping. The treatment can cover the entire structure or just part of it [97, 98]. For example, in reference [99] Yildiz and Stevens found that the increase of the viscoelastic layer thickness can increase the damping by as much as 100%. However, the presence of the viscoelastic layer degrades the stiffness and strength performance of the sandwich. To overcome this issue Chung [30] proposed the addition of carbon micro-filament at the interface between the soft and the stiff layer, which also provides a better resistance to temperature. Instead of a soft core, Youzera et al. and Bothelo et al. [100, 101] used carbon/epoxy and glass/epoxy layers constrained by metal layers, correlating theoretical and experimental results and investigating the effect of the composite layer orientation, relative thickness of core and face sheets and aspect ratio of the beam respectively.

3.4. Advanced hybrid composites

In order to increase the energy dissipated by composite structures, hybridization with highly damped materials or systems has been proposed [102]. Examples of such hybrid composite structures are composites with Shape Memory Alloys (SMA) inserts and Piezoelectric (PZT) patches. In both cases active vibration control is possible.

SMA hybrid composites

SMA Hybrid composites exploit *Superelasticity* (SE) and the *Shape Memory Effect* (SME) [103]. Both phenomena are the consequence of the existence of two material phases: martensite, which is stable at low temperature, it has low Young Modulus and high damping coefficient, and austenite, which is stable at high temperature, it has high Young Modulus and low damping coefficient. The SE is associated with the pseudo-elastic behaviour that the alloy shows in the austenitic phase and thanks to which apparent plastic strains are recovered upon unloading. The SME occurs when the plastic strains of the martensitic phase are recovered upon heating by inducing a martensite-to-austenite phase transformation. Both phenomena results in high internal friction which dissipates energy.

The difference in stiffness and hysteresis that results from the phase change can be used to alter the dynamic characteristics of the host structure. SMA can be introduced in composite structures in different forms, each implying a different activation mechanism of the SE or SME.

In reference [104] a two-layer beam has been tested, one ply being made of pure epoxy resin and one ply containing dispersed short Ni-Ti fibres in epoxy resin. The effect of SMA fibre volume fraction and temperature on the logarithmic decrement is investigated: increasing the temperature leads to an increase in the logarithmic decrement which, at given temperature, is higher for higher volume fractions. The effect of the SMA is also visible on the Glass Transition Temperature of the SMA composite, which shifts towards a lower value as the SMA volume fraction increases. The use of long fibres is reported by Baz [105] where the active control by means of electrical activation of SMA fibres is studied. Attenuation of vibrations is achieved by selective or full activation of SMA fibres which modify the stiffness of the component and shifts the natural frequency away from the excitation one. In reference [106] Baz warns that electrical activation of Nitinol fibres has as side effect the increase of

temperature which eventually can soften the polymer matrix thus increasing the overall damping but decreasing the stiffness.
Other authors, such as Ameduri et al.[107], Diodati and Ameduri [108], Rogers [109], Wei et al. [110] and Zhango and Zhao [111] have investigated the use of SMA for active/passive vibration suppression in composite structures proving their effectiveness. Other reinforcement forms are possible such as woven fabrics [112] and ribbons [113]. In reference [114] the passive damping capacity of SMA composites are assessed thoroughly by investigating, beside temperature and stress levels that activate SE and SME, the effect of pre-strain. Pre-strain levels affect the relative percentage of phases in the SMA and, given their different damping properties, the amount of energy dissipated during vibration; in particular a higher pre-strain led to a higher damping.
Despite the promising results, many difficulties exist that prevent from full exploitation of SMA hybrid composites [109–111, 114]: the temperature and stress-dependent characteristics of SMA are not constant in time and showed degradation after several thermal cycles; thermal effects associated to the active control of SMA degrade matrix stiffness (though they increase damping); differences in thermal expansion coefficient between SMA and host material may induce damage and/or debonding.

PZT hybrid composites
Piezoelectric patches are added to composite or sandwich structures for sensing and actuation purposes. The piezoelctric effect is such that a voltage applied to the PZT is transformed into strain of the PZT patch and a strain is transformed into a voltage. Piezoelectric patches are appealing in the field of active vibration control. With respect to SMA, PZT sensors show a faster response at the expense of smaller displacements [110]. Though mainly used in the form of thin films, the use of PZT fibres coated by an electrode is reported by Gibson [71] and the use of rods is reported by Arafa and Baz [115]. In reference [116] the interaction of active and passive vibration control is investigated and not only is the stiffness of the PZT patch accounted for but also its damping as well as the dissipative properties of the adhesive. Comparison of numerical and experimental results highlights an increase in accuracy when these effects are included in the model. By properly tuning the gain of the control system the damping ratio increases by a factor 2. In a series of papers by Moita and co-workers [82, 117–122] the modelling of PZT hybrid composites is thoroughly investigated showing the capability of active control systems for different laminations schemes, loading conditions (free and forced vibration) and boundary conditions. In all cases significant improvements have been obtained.
In reference [123] the piezoelectric mechanism coming from extensional and shearing deformation has been controlled separately or in a combined form to provide damping.

4. Damping models

4.1. Linear viscoelastic models

Damping models that have been proposed for composite materials stem from models for viscoelastic materials. The main reason is that most of the dissipation comes from the polymeric matrix. The simplest way to represent damping is the use of linear viscoelastic models, such as Kelvin-Voigt, Maxwell or Zener models. The main advantage of such models is the low number of parameters needed to characterise the viscoelastic behaviour of the material, the ease of their estimation and their availability in commercial finite element codes. They are however simplistic and fail to fully represent the physics underlying the mechanisms of energy dissipation. The Zener model in particular is based on thermoelastic considerations. When the period of vibration is smaller than the relaxation time, heat flow cannot occur and other phenomena, which are not modelled, become dominant [23]. Improvements can be achieved by the use of the Prony series, which describes damping as a summation of decaying exponentials: the more terms are added to the summation the higher the accuracy, although this implies extensive experimental tests to determine the different time constants [124].

4.2. Complex modulus and loss factor

The complex modulus approach is extensively used especially in finite element codes. The material stiffness is given by a complex number with the real part (*storage modulus*) referring to the elastic behaviour and the imaginary part (*loss modulus*) referring to the dissipative behaviour. The ratio between the two gives the loss factor of the material.

The earliest studies on the effectiveness of the complex modulus to represent damping date back to 1962 thanks to Neumark [125], who reviewed the use of the complex stiffness in the case of forced and free vibration. Hashin [126, 127] developed an expression for the complex modulus of inhomogeneous viscoelastic materials. The theory showed some shortcomings for high volume fractions of spherical inclusions and fibrous reinforcements.

The accuracy and physical meaning of the complex stiffness were discussed by Crandall [128, 129] who addressed the issue of non-causality that arises when a constant loss factor is chosen. The impulse response of systems with frequency-independent loss factors do not vanish before the application of the impulse. Pritz [130] observed that the use of a constant loss factor is in contrast with the observed increase in damping with increasing frequency; he also observed that most experimental data in logarithmic scale can be interpolated by straight lines. He therefore suggested a power law to express the relationship between the loss factor and the loss modulus of materials based on Kramer-Kronig dispersion relations to ensure causality. The method is limited to a finite bandwidth and, under these conditions, most materials exhibit a nearly constant loss factor.

The loss factor is most often used for hysteretic damping, being associated with the amplitude of the hysteresis loop obtained by cyclic loading of the material. If the excitation stays below the proportionality limit, that is in the linear region of the stress-strain curve, the hysteresis loop is approximated by an ellipse. When the proportionality region is exceeded, the loop deviates from the elliptic shape and dissipated energy is given by $\int_0^{\varepsilon_0} \sigma d\varepsilon$; non-linear models have been proposed to consider both low and high stress regions but their mathematical complexity make them unattractive [131].

4.3. Strain-energy methods

In composite materials dissipated energy is the result of both matrix and fibre contributions. Ungar and Kerwin [132] defined one of the earliest relation for the determination of loss factors in composites, in which contributions are divided based on the energy stored in constituent materials. Some years later Adams and Bacon [47] postulated that the dissipated energy can be decomposed and associated with the principal stress components; the ratio between the stored and the dissipated energy, along each principal direction, gives the *Specific Damping Capacity* (SDC). They derived the SDC expression from the free flexure of beams along the x-direction while Ni and Adams [133] considered the case of an applied bending moment. The SDC final expression will differ in the two cases though a common general form can be written; only the in-plane stress components were accounted for; Saravanos and Chamis [134] gave SDC expressions for all the six stress components in both on-axis and off-axis loading conditions. In reference [135], Kalisake and Rothert use the SDC in combination with the *Method of Cells* originally proposed by Aboudi [136] in order to find a damping matrix in terms of the SDC; for each principal stress component a SDC value is calculated. The assumptions of uncoupled modes, which applies only to certain modes, ensures that the SDC matrix is diagonal. These methods are based on strain energy considerations and can be easily combined with finite elements or other approximation schemes, like the Rayleigh-Ritz method [68], though in this case only simple structures can be effectively analysed. In this framework, the *Modal strain energy* (MSE) method and the *Iterative Modal Strain Energy* (IMSE) method deserve mention. Firstly proposed by Johnson et al. [137], the MSE calculates the modal loss factor starting from the strain energy of the undamped configuration; the IMSE allows to take the frequency dependence of the complex modulus into account [138]. However, the underlying hypotheses constitute the main limit of the two methods which assume that no changes occur in the damped mode shapes, restricting the application only to lightly damped structures. Santosa and Symes [139] proposed an homogenization technique for the loss factor of composites with anisotropic dissipative behaviour that could take into account the time-dependence of the visco-elastic properties. In the attempt to give a more physical representation of damping, Lazan [140] and Kume et al. [141] proposed a power law function to relate stresses and damping. Still based on strain energy calculations, the method is approximate and give acceptable results for lightly damped structures and in the linear region of the hysteresis loop (low stress). No indication is given on how to extract the necessary material parameters from experiments. Gounaris et al. used the same theories in an iterative numerical-experimental scheme [142, 143]. White and Abdin [144] used a power law function based on Cox and McLean and Read model to calculate the SDC of short fibre reinforced polymers.

4.4. Physics-based models

Other models try to account for the physics of energy dissipation. All the models can be gathered under the definition of *internal variables models* because they require additional internal variables to define damping. Enelund and Lesieutre [145] and Enelund and Olsson [146] used an *Anelastic Displacement Field* (ADF) together with fractional derivatives; the same model has been used by Wang and Inman [147] and implemented in a finite element scheme. They also reported the Golla-Hughes-McTavish model which combines a standard Maxwell model with internal dissipation variables. Internal thermodynamic variables are chosen by Lesieutre [148] who developed the *Aug-*

menting Thermodynamic Field (ATF); Dovstam [149–151] used the ATF as a starting point for the development of the *Augmented Hooke's Law* (AHL) based on internal free energy density functions. All these models are theoretically able to describe the dissipation phenomena at the desired level of physical accuracy and complexity the dissipation phenomena but they are not fully supported by experimental techniques that allow to assess their contribution.

5. Conclusions

The literature currently available on damping in composite materials has been reviewed. Several damping models are available but choosing the correct one requires some preliminary considerations:

- complex modulus, standard linear models, Ungar-Kerwin and Adams-Bacon strain energy approaches are easy to handle; input parameters can be easily found from standard creep and vibration tests. However, they only provide an estimate of material damping and fail to represent the underlying physics. In some cases, if the problem configuration changes new damping values should be found;

- the internal variables approaches are based on relations between damping and physical phenomena such as internal thermodynamic or anelastic displacement fields. Once these relations are established, the analytical models can also be used to predict changes in damping values due to a change in the problem definition. However, there are no documented and reliable experimental procedures to find such relations.

A wise choice would be to always prefer models whose parameters can be easily found and whose validity can be experimentally checked. If simulation is the main application, then models that can be easily integrated in finite elements schemes should be given priority.

Berthelot et al. [13] proved for example that the use of a strain energy approach can be easily integrated into finite element schemes and eventually applied to several types of composite structures. Combining the loss factors deduced from beam specimens testing with the strain energy information gained from finite element analysis it is possible to obtain good estimations of the dissipation taking place in the structure as a whole. Design of composite materials is strictly case dependent which on the one hand allows for a high level of optimization and on the other hand prevents the definition of generally valid rules and relations such as the ones available for metals. All constitutive parameters play a role both at material level and system level. Complex relations exist among factors such as ply angle and volume fraction, stack up and type of constraints; many experimental data have been presented that could be used to define empirical relations and guidelines for similar problem configurations;

Advanced hybrid materials proved to be a feasible design solution; even though their cost may increase, the gain in performance is evident. Nano-scale reinforcements are appealing as well, but an additional effort has to be made to gain knowledge of the mechanisms behind their high dissipation potential. Low cost solutions can be found in the domain of sandwich and sandwich-like structures which are quite spread in many application fields.

Experimental characterization of composites dynamic properties is still far from being standardized. Moreover, results seem to be setup-dependent meaning that traditional methods introduce non-negligible sources of damping. This is particularly the case for air damping, friction at clamps and mass effect of contacting excitation and measurement devices. Contactless sources and probes that employ acoustic and optical methods have become in the last years minimum setup requirements increasing the reliability and repeatability of results.
Further research efforts are needed on the topics outlined above, in order to reach a better understanding of the physics underlying the energy dissipation mechanisms. This can only be addressed by experimental characterisation. The improvement of existing techniques or new approaches are therefore needed to overcome current shortcomings. Finally, the translation of physical observations into mathematical terms should be addressed to improve the capabilities of predictive models.

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