Modelling and optimization of least-cost water distribution networks with multiple supply sources and users

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Abstract: The proper allocation of water resources is a very important practical problem in the field of water network planning. Optimization models that are expeditious and easy to use for all stakeholders of the sector play an important role for water resource management. The present work resumes and reviews a least-cost optimization model proposed by our group [1], able to design a water distribution network with multiple supply sources and multiple users. This approach requires of solving an optimization problem based on a nonlinear objective function which is proportional to the cost of the water distribution network. The cost of pre-existing pipelines is considered null. A more realistic scenario, able to consider the maximum flow rate allowed for existing sources-users connections, is considered here. In order to illustrate the usefulness and flexibility of the proposed approach, an application of the model to the real case of the province of Croton, Southern Italy, is presented.

Keywords: Mathematical modelling, non-linear optimization, water management, sustainability

1. Introduction

Mathematical models represent the key tools to solve the multitude of problems encountered in the different fields of physics and engineering [2-8]. Since the 1960s, in the particular area of water resources management, many complex problems have been addressed from the point of view of the system analysis and general optimization techniques [9-12]. Over the years, many optimization models have been developed and applied in multi-use reservoir operation [13], irrigation water management [14], design and management of water distribution network [15], water resource allocation [16-20]. The state-of-the-art of these models has been reviewed by several research groups [21-26].

One of the most challenging problems in water resources management is represented by the optimization of water-supply network. The pursuit of optimal management of these systems requires the improvement of reliability or operational efficiency [27-29], risk analysis associated with the vulnerability of drinking water systems [30], the increase in water availability through, for example, the use of non-conventional water resources [31-33], the proper allocation of available water resources [1]. Due to the presence of multiple water sources and different types of conflicting water users, the optimal allocation of water resource is a complex problem [34-35] and many studies have been carried out on this topic with applications also to real cases [18,36,37]. Moreover, the concept of sustainable management of water resources [38] includes not only the ability to ensure proper supply of the resource, but even the ability to organize a distribution system with high efficiency and low cost [39]. Different types of optimization techniques have been proposed to deal with the problem of water distribution network design.

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Linear Programming was among the first optimization method applied by many researchers in water resources management [40, 41]. The main limitation of this approach lies in the non-linearity of many water distribution system design problems. Therefore, other researchers proposed the use of Nonlinear Programming [42, 43] or Dynamic Programming [44] techniques for the optimization of pipes-network. A different approach is based on the implementation of stochastic optimization models [26, 45, 46]. Methods used in the framework of water resource management include Genetic Algorithms [47, 48, 49], Simulated Annealing [50], Harmony Search Optimization [51], Ant Colony Optimization [52, 53], Shuffled Frog Leaping Algorithm [54], Memetic Algorithms [55], Particle Swarm Optimizations [56], Agent Swarm Optimization [57], just to cite some. In [58] the evolutionary algorithm Differential Evolution, an improved version of Genetic Algorithm, is applied to New York water supply system pipe sizing problem. Mixed approaches were also developed. For example, a combination of Genetic Algorithm and Linear Programming for solving water distribution system design problems is described in [59], whereas Genetic Algorithm and Integer-Linear Programming are linked in a hybrid optimization scheme proposed in [60]. In [61] a modified Harmony Search Algorithm incorporating Particle Swarm concept is proposed. In [62] authors propose a hybrid particle swarm optimization and differential evolution method, linked to the hydraulic simulator, EPANET, for minimizing the cost design of water distribution systems. In [28] nonlinear programming techniques are coupled with existing water distribution simulation models.

Even in the case of drinking water distribution systems, the proper allocation of available resources is an important aspect in order to achieve an optimal management of such type of systems and different studies have been conducted in this field with different approaches and conceptual models. [63] presents a water resources allocation model, which takes into account economic and socio-environmental factors, developed and applied to serve as a multi-sectoral decision for water resources management. In this model, the correct allocation of the water resource is analysed to minimize global system cost. [47] proposes an optimization model for the optimal planning of complex water systems with multiple supply sources and multiple users, taking into account environmental considerations. This model analyses the water resources demand in different periods and areas and formulates a sustainable development water resources allocation. In [64], authors developed an optimal model for water allocation and water distribution network management in which cost and water conservation are regarded as objectives. [65] investigates the optimal allocation of water resource for urban water management system through a water resource optimal allocation model based on multi-agent modelling technology, where different optimization objectives are abstracted into various properties of different agents. In this model, water quality, transportation price, economic benefit, ecologic benefit are used as adimensional parameters to ensure better operating conditions.

In this context, the present paper resumes and develops a least-cost optimization model, relative to proper allocation of water resources for drinking water, proposed in [1]. This procedure analyses the water resources availability on a given territory with respect to the water demand and calculate an idealized water distribution network able to achieve the optimal allocation in terms of minimum overall cost for the distribution system construction. This approach requires of solving an optimization problem based on a nonlinear objective function which is proportional to the cost of the water resources transferring. The formulation of the model is not complex in terms of type and number of input data, and in terms of mathematical modeling. As a matter of fact, the proposed optimization model does not require the collection and processing of a large and complex amount of data and, compared to other models, does not require excessive computational costs. The model is well applicable to different territorial scales and can also be easily adapted to different types of distribution systems.

Two situations are considered: in the first case, the optimal water distribution system is calculated in the hypothesis that not connections initially exist among source and user nodes; in the second one, starting by a real distribution system, the minimal changes are determined in order to optimize the water allocation. The developed optimization model was implemented in MATLAB [66] through the use of the function “fmincon” which allows an accurate and expeditious computational evaluation of

the optimal solutions. The usefulness and flexibility of the proposed procedure is illustrated on a real case study in province of Croton, Southern Italy.
The remainder of this paper is organized as follows. The proposed methodology and the optimization model are presented in detail in Section 2, while the corresponding implementation is described in Section 3. Results of the case study are summarized in Section 4. Finally, conclusions are presented in Section 5.

2. Optimization model

An optimization model is a set of equations that define and relate all of the components in a decision problem. To represent a decision problem as an optimization model it is necessary to define the decision variables and the uncontrollable variables (or input data). The first ones describe possible decisions to be made, the second ones represent factors determined by the system environment that affect the result but are not under the control of decision makers. The performance of a given choice of the decision variables is evaluated by a suitable objective function, that is a measure of how well the proposed goal is accomplished. Design choices and uncontrollable parameters can introduce limitations to the values of the decision variables. Appropriate constraints then specify, for a given problem, the values for which the decision variables have meaning and all possible restrictions, requirements, and interactions that could limit the values of the decision variables. Therefore, the output of an optimization model represents the best choice of the decision variables to achieve the best measurable performance of the system in term of minimization/maximization of the objective function under given constraints. The choice of a specific objective function and the design constraints determine the modeling approach of a given decision problem.

2.1 Model formulation

The proposed methodology is conceptually inspired by the classic Transport Problem of Operation Research, so called because many of its applications relate to the problem of finding the best way to carry goods [67]. In fact, the transport problem is an issue of optimal management of the amount of materials that must be transferred from many places where they lie in other places with the aim of minimizing transport costs (time). This problem has been particularized to the case of water resources in order to rationalize the use of available volumes within a territory, integrating the availability and the needs [1]. In the case of the water resources, the transfer cost of the water is related to several factors, and in general is not expressed in terms of unit cost directly proportional to the flow, but in terms of overall cost per unit length. Therefore, the problem that has been obtained is no longer linear, but is a non-linear constrained problem.
The problem dealt with in this paper is defined as follows:

- m source nodes (springs, wells, derivations) are present in a given territory, and the water availability (l/s) of each source is labeled as a_i, $i = 1,2,\dots,m$;
- there are n destination nodes (users), and the demand of each user (l/s) is labeled as b_j, $j = 1,2,\dots,n$;
- Q_{ij} is the quantity of water (l/s) transferred from the source node i to the destination node j;
- C_{ij} represents the cost of the transferring of Q_{ij};
- the water resource allocation task is to decide the optimal allocation configuration which minimizes the total cost of the whole water distribution system:

$$C = \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} \quad (1)$$

The specific modeling of the cost function defines the adopted schematization and identifies the basic information necessary to design the optimization problem.

2.2 Objective function

According to the approach proposed in [1], the cost of the pipeline joining the source i to the user j can be expressed by means of the monomial formula of Contessini [68], which is more appropriate for the characteristics of the existing pipelines in the area of the case study:

$$C_{ij} = K \frac{0.0012^{\frac{\alpha}{5.26}} \cdot L_{ij}^{\left(1+\frac{\alpha}{5.26}\right)}}{Y_{ij}^{\frac{\alpha}{5.26}}} Q_{ij}^{\frac{2\alpha}{5.26}} \quad (2)$$

where L_{ij} is the distance between nodes i and j, and Y_{ij} is the corresponding piezometric head difference. Therefore, the cost of the whole water distribution system that defines the optimization model is

$$C = \sum_{j=1}^{m} \sum_{i=1}^{n} \left(K \frac{0.0012^{\frac{\alpha}{5.26}} \cdot L_{ij}^{\left(1+\frac{\alpha}{5.26}\right)}}{Y_{ij}^{\frac{\alpha}{5.26}}} Q_{ij}^{\frac{2\alpha}{5.26}} \right) \quad (3)$$

where K and α depend on the material of the pipeline and, for the characteristics of the existing pipelines in the area of the case study, the value $\alpha \sim 1$ is assumed [1], while the parameter K can be left in parametric form because it does not influence the minimum cost configuration. The solution of the optimum water allocation problem requires of determining the decision variables Q_{ij} such that the cost function (3) is minimized and the appropriate constraints, described below, are satisfied. The desired solution is the set of links between source nodes and destination nodes, which represents the proper allocation of water in the area under study.

In order to simplify the collection of experimental data required to apply the proposed approach to a real case study, L_{ij} and Y_{ij} in (3) were calculated as Euclidean distance between the points that identify the position of the nodes i and j. With this approximation, the knowledge of the geographic positions of the source and destination nodes is sufficient to completely define the input data of the model. Therefore, to carry out a simulation of a real case study, it has been necessary to identify the location of all water intake structures present in a given area and collect the information about the water availability. In addition, for each municipality in the area was calculated the center of gravity of the polygon representing the administrative boundaries, which was taken as the position of the destination node. Finally, it has been estimated the resident population and fluctuating population in order to calculate the water demands. It is worth noting that, due to the above-mentioned approximation, the minimum-cost water distribution system obtained from the optimization procedure does not represent a physically feasible hydraulic infrastructure but a sort of sketch-map of the connections diagram. However, the presented optimization model retains its conceptual value regardless of this simplifying choice.

2.3 Constraints

Constraints are physical and planning limitations imposed on the model to represent the actual operational characteristics of a given water resources system. With respect to the investigated problem, the following considerations can be drawn.

The total water supplied from each source node i cannot exceed the maximum water supply capacity of the source:

$$\sum_{j=1}^{n} Q_{ij} \leq a_i, \; i = 1, \dots, m \quad (4)$$

In order to obtain proper water allocation, the water demand of each user node j must be fulfilled and the water resources balance between water supply and water demand constraint is written as

$$\sum_{i=1}^{m} Q_{ij} = b_j, j = 1, \dots, n \quad (5)$$

The water quantity must satisfy

$$Q_{ij} \geq 0, i = 1, \dots, m; \text{j} = 1, \dots, n \quad (6)$$

Finally, to exclude pump stations, the following constraint is imposed

$$Q_{ij} = 0 \text{ if } Y_{ij} = h_i - h_j \leq 0 \quad (7)$$

Note that, in the case where demand exceeds supply, it is necessary to introduce a dummy source with assigned supply and proper cost

$$\mathrm{a_{dum}} = \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{b_j} - \sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{a_i} \quad (8)$$

The model thus defined determines the optimal distribution of the water resource present on a given territory. To take into account possible water distribution systems already present in the territory, it is necessary to consider further constraints in the model. In particular, the cost C_{ij} is set to zero if the source node i is connected to the destination node j by an existing pipeline

$$C_{ij} = 0, \forall \text{ connected } i, j \quad (9)$$

However, the possible flow rate Q_{ij} between already connected nodes i and j, cannot exceed the maximum value Q_{ij}^{M} allowed by the existing pipeline

$$Q_{ij} \leq Q_{ij}^{M}, \forall \text{ connected } i, j \quad (10)$$

If available information allows to know only the total flows that can be supplied to a user node j by N_j subsets of its existing connections, the constraints on the actual flow rates for j can be expressed by the following expressions

$$\sum_{i \in \mathbb{I}^s(j)} Q_{ij} \leq Q_{max}^{s}(j), s = 1, \dots, N_j \quad (11)$$

where $\mathbb{I}^s(j) = \{i_1, \dots, i_{n_s}\}$ identifies the s[th] subset composed by n_s already connected to j source nodes, and $Q_{max}^{s}(j)$ is the maximum total flow allowed from this subset.

3. Implementation of the optimization problem

In order to solve the non-linear optimization problem with the constraints under investigation, a MATLAB code was developed. The "fmincon" function of the optimization toolbox of MATLAB was then used to find the optimal minimum cost set of Q_{ij} amounts. The "fmincon" routine uses a sequential quadratic programming (SQP) technique representing a powerful nonlinear programming method [69, 70]. In more details, "fmincon" is able to find a vector $x = (x_1, x_2, \dots, x_N)$ thath is a local minimum of a nonlinear scalar function $f(x)$ subject to the constraints on the allowable x. Among other available options for "fmincon", linear constraints and linear inequalities are expressed as

$$A \cdot x = v \tag{12}$$

$$B \cdot x \leq d \tag{13}$$

and it is also possible to set a lower and upper bound on the design variables, so that the solution is always in the range

$$l_b \leq x_k \leq u_b,\ k = 1, \dots, N \tag{14}$$

Therefore, in order to use the "fmincon" function, the decision variables Q_{ij} of the optimization problem presented in the previous section are rearranged into a single vector of length $N = n \cdot m$

$$x = (Q_{11}, Q_{12}, \dots, Q_{mn}) = (x_1, \dots, x_k, \dots, x_N) \tag{15}$$

where

$$k = k(i, j) = (i - 1) \cdot n + j \tag{16}$$

whit $1 \leq i \leq m$, $1 \leq j \leq n$, and the cost (3) is rewritten as a nonlinear function of x

$$C(x) = \sum_{k=1}^{N} \left(K \frac{0.0012^{\frac{\alpha}{5.26}} \cdot L_k^{\left(1+\frac{\alpha}{5.26}\right)}}{Y_k^{\frac{\alpha}{5.26}}} x_k^{\frac{2\alpha}{5.26}} \right) \tag{17}$$

Moreover, it is necessary to rewrite the constraints (4-6) and (9-11) in the form (12) and (13) by appropriately building the matrices A and B. In particular, it is possible to put constraints (5) in the form $A_1 \cdot x = v_1$ with

$$v_1 = (b_1, b_2, \dots, b_n)' \tag{18}$$

and A_1 defined as a $n \times N$ block matrix formed by m identity matrices $n \times n$

$$A_1 = \left(\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \ \cdots \ \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \right) \tag{19}$$

Constraints (7) are in the form $B_1 \cdot x \leq d_1$, where

$$d_1 = (a_1, a_2, \dots, a_m)' \tag{20}$$

and B_1 is a $m \times N$ matrix formed by m blocks, where the k^{th} block is an $m \times n$ matrix with all elements null except the k^{th} row consisting in a $1 \times n$ unitary vector

$$B_1 = \left(\begin{bmatrix} 1\ 1 \dots 1 \\ 0\ 0 \dots\ 0 \\ \vdots \\ 0\ 0 \dots\ 0 \end{bmatrix} \begin{bmatrix} 0\ 0 \dots\ 0 \\ 1\ 1 \dots 1 \\ \vdots \\ 0\ 0 \dots\ 0 \end{bmatrix} \ \cdots \ \begin{bmatrix} 0\ 0 \dots\ 0 \\ 0\ 0 \dots\ 0 \\ \vdots \\ 1\ 1 \dots 1 \end{bmatrix} \right) \tag{21}$$

Also constraints (7) and (10, 11) can be expressed in the form $A_2 \cdot x = v_2$ and $B_2 \cdot x \leq d_2$ respectively, but they in general do not give rise to matrices with regular structure. In effect, A_2 and B_2 depend on the orography of the territory, on the topology of the existing water distribution network and on the awareness about maximum flows permitted in the network.

In more details, if $N_{\Delta Y}$ is the number of constraints (7), A_2 is a $N_{\Delta Y} \times N$ matrix and the generic row has all 0 values and a single 1 in the position $k(i, j)$ corresponding to the flow $x_k = Q_{ij}$ for which holds the (7), whereas $v_2 = (0,0, \dots 0)'$ is the null vector of length N.

Therefore, for the case under study, the elements in the expression (12) are

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{22}$$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{23}$$

With respect to the existing water distribution network, if there are N_e connections, the constraints (10) correspond to a $N_e \times N$ matrix B_2, whose generic row has all 0 values and a single 1 in the position $k(i,j)$ if nodes i and j are connected, whereas $d_2 = (Q_1^M, \ldots, Q_{N_e}^M)'$ is the vector containing the maximum flux values allowed in the existing pipelines.
Analogously, the constraints (11) correspond to a $N_{ss} \times N$ matrix B_2, where N_{ss} is the number of total subsets of the water distribution network for which the maximum allowed flux is known. The generic row of B_2 has all 0 values except in the positions $k(i,j)$ with $i \in \cup\, \mathbb{I}^s(j)$, where there is a 1. The vector d_2 contains the corresponding total set of the known $Q_{max}^{(s)}(j)$ values. The (13) is expressed by means of

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{24}$$

$$d = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \tag{25}$$

Finally, the lower bound for the (14) is given from condition (6), whereas the upper bound is equal to a_i for each $x_k = Q_{ij}$. Reformulating in this way the optimization problem, the function "fmincon" provides the optimal allocation of water resources.

4. Case study

In this section, the results of the proposed procedure applied to the real case study of the province of Croton, Southern Italy, are shown. As mentioned above, it has been necessary to collect all the data related to springs, derivations and wells in the area of study, and all the data related to the demands of users within the territory itself. Regarding the sources, in this area there are 29 springs, 3 derivations and 7 wells for a total of 1534.8 (l/s) drinking water availability. The water demand by municipalities is 922.2 (l/s). Therefore, in this case the water availability exceeds the demand but the water distribution is not balanced and, especially in summer, several municipalities are affected by shortage of water. For this reason, the province of Croton is well suited to apply the optimization procedure. The problem is characterized by $m = 39$ source nodes and $n = 27$ destination nodes, and $N = m \cdot n = 1092$ decision variables Q_{ij} have to be determined in order to minimize the cost (3).
A first application of the proposed procedure concerned the optimization of the water allocation without considering the existing water distribution system. The obtained results have shown compatibility with the existing water distribution systems but since the total availability of water resources exceeds requirements, the proposed procedure has identified a solution that:

- significantly reduces the number of connections between water intake structures and users;
- does not use some water intake structures characterized by low water flow rate (actually, these water intake structures are connected to single municipalities).

A graphic representation of the obtained results is shown in Fig. 1 in which:

- open triangles represent not used sources;
- filled triangles represent source nodes whose water is completely distributed to users;
- filled square indicate source nodes connected also with the dummy destination (source node with a surplus of water that remains at the source node itself);

- filled magenta circles represent the destination nodes;
- black solid lines represent found connections between source and user nodes existing also in the real water distribution system;
- dash-dotted green lines indicate new connections.

In order to satisfy those users that have a water supply lower than their needs and to obtain a more efficient water distribution system, a second optimization process has been performed considering the existing pipeline. Also in this case, the model identifies a solution that can be considered effective in terms of distribution of the water resource and, through a redefinition of some water schemes, the obtained solution allows to meet the needs of all utilities.

A graphic representation of the results obtained is depicted in Fig. 2, where:

- dash-dotted green lines indicate new connections;
- filled triangles represent source nodes whose water is completely distributed to users;
- filled square indicate source nodes connected also with the dummy destination (source node with a surplus of water that remains at the source node itself);
- filled magenta circles represent the destination nodes;
- black solid lines represent pre-existing connections.

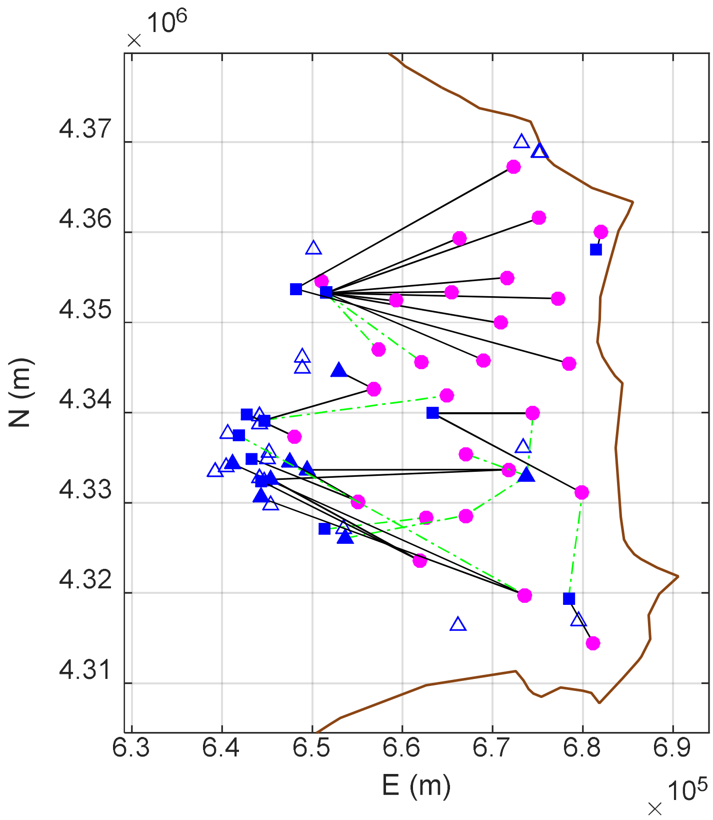

Figure 1 Graphical representation of the water distribution links obtained from the proposed optimization model applied to the case study of Croton, southern Italy. Considered hydraulic information corresponds to real data in terms of water demand/availability, but pre-existing pipelines are not taken into account. Triangles and squares denote source nodes, circles represent user nodes. Dash-dotted green lines indicate new connections with respect to the existing water distribution network.

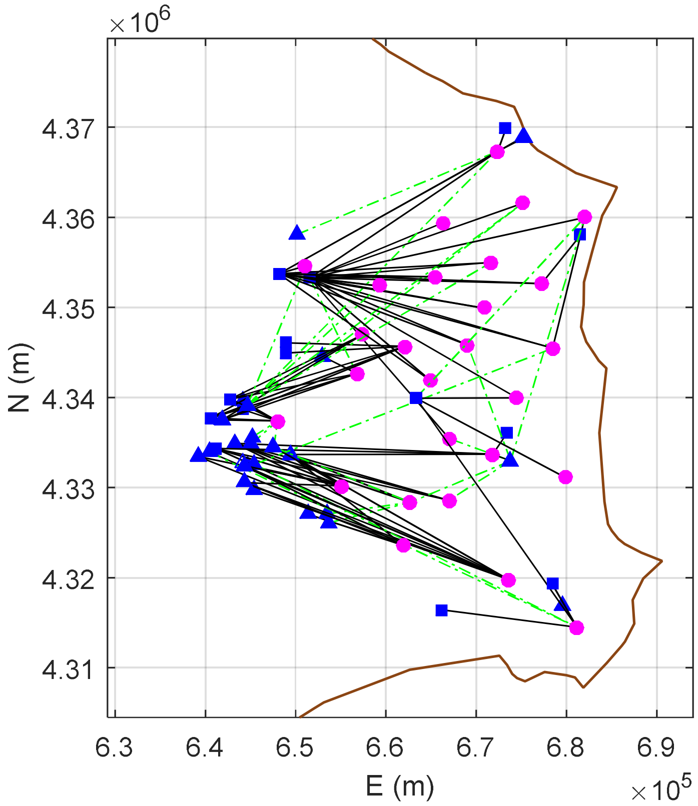

Figure 2 Graphical representation of the water distribution links obtained from the proposed optimization model applied to the case study of Fig. 1, but considering the existing water distribution network. Dash-dotted green lines indicate new connections found from the optimization process in order to satisfy the water requirement of all users.

It is worth noting that other cost-based allocation models link the cost to the economic and social issues [65] or to the economic, social and environmental issues [47, 63]. These models introduce an intrinsic goal related to the sustainability in water resource management (economy, society and the environment are the three pillars of sustainable development), linking the assessments to well defined time scales. It is clear that [47, 63, 65] require many and complex data inputs related to economic benefits (e.g. the benefit coefficients of unit water supply to user) and to characteristic properties of natural and social ecosystems (e.g. water demand for users rated for types of utilization, pollutant content in user wastewaters). On the other hand, the proposed least-cost optimization model offers a simpler and more expeditious cost evaluation in a water distribution network based on the easy-to-find hydraulic variables.

5. Conclusions

The rational and sustainable management of water resources requires the analysis of the existing water systems and the identification, through optimization models, of solutions capable of balancing the distribution of resources also through the redefinition of the water systems.
This paper describes a least-cost optimization model for the allocation of drinking water resources that is easy to use both in relation to the input data and in terms of mathematical modeling. The proposed approach uses a MATLAB code making expeditious the implementation of the model itself.
The practical relevance and the potential applications of the described model are related to the possibility to solve local problems of supply, through the redefinition of the existing water systems, the transfers of resources and the creation of the necessary works to supply.

In order to illustrate the usefulness and flexibility of the proposed approach, an application of the model to the real case of the province of Croton, Southern Italy, is presented. This case study demonstrated the reliability of the model, which results applicable to real data. However, the water distribution system obtained from the proposed procedure not always represents a physically feasible hydraulic infrastructure. The output of the model provides a first indication of the optimal connections needed to cover the water demand of a given territory. From this first level of detail it may be possible that some connections indicated by the model are not feasible, for example due to the complex orography or legal and hydrogeological constraints. This first level of the proposed model can be specified with successive levels of detail taking into account the territory constraints in the evaluation of the sources-users distances. Therefore, the proposed model constitutes, at the moment, an expeditious procedure, with low computational cost and which requires a minimum amount of information. Future model improvements will involve additional constraints leading to more realistic results.

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