Model predictive control for constrained networked systems subject to data losses

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Abstract

The paper addresses the stabilization problem for constrained control systems where both plant measurements and command signals in the loop are sent through communication channels subject to time-varying delays and data losses. A novel receding horizon strategy is proposed by resorting to an uncertain polytopic linear plant framework. Sequences of pre-computed inner approximations of the one-step controllable sets are on-line exploited as target sets for selecting the commands to be applied to the plant in a receding horizon fashion. The communication channel effects are taken into account by resorting to both Independent-of-Delay and Delay-Dependent stability concepts that are used to initialize the one-step controllable sequences. The resulting framework guarantees Uniformly Ultimate Boundedness and constraints fulfilment of the regulated trajectory regardless of plant uncertainties and data loss occurrences.

Key words: Networked control systems; Receding horizon control; systems with time-delays; Uncertain polytopic models

1 Introduction

Networked Control Systems (NCS) represent the interconnection of a set of plants equipped with sensing and communication devices. From an abstract point of view an NCS can be regarded as a system comprised of the plant to be regulated and of actuators, sensors, and controllers, coordinated via a communication channel. NCS stability analysis and control design features are attracting considerable attention in the technical literature, see (Hespanha, Naghshtabrizi & Xu 2007, Montestruque & Antsaklis 2004, Franzè, Famularo & Tedesco 2011) and references therein. Recent contributions and tutorials on NCS modeling and performance analysis have been conducted using discrete-time (Cloosterman, van de Wouw, Heemels & Nijmeijer 2009), sampled data (Fridman & Shaked 2005) and continuous time (Heemels, Teel, van de Wouw & Nesic 2010) framework approaches, respectively.

Of interest here are constrained Receding Horizon Control strategies which are an extremely appealing methodology for NCS stabilization due to their intrinsic capability to generate, at each time instant, a sequence of virtual inputs which can be transmitted within a single data-packet (Quevedo, Silva & Goodwin 2007). Noticeable contributions on this matter are from (Muñoz de la Peña & Christofides 2008, Quevedo & Gupta. 2013, Pin & Parisini 2011): in (Muñoz de la Peña & Christofides 2008), the authors consider a receding horizon strategy for nonlinear networked systems under wireless and asynchronous measurement sampling. Gupta and Quevedo (Quevedo & Gupta. 2013) extend a control scheme for nonlinear plants, popular in real-time systems, to tolerate the presence of time-varying processing resources (such as variable delays, packet losses/drops etc.), known as *anytime* algorithm. In (Pin & Parisini 2011), following the same ideas as (Muñoz de la Peña & Christofides 2008), a nonlinear RHC scheme exploiting a Network Delay Compensation strategy is proposed to efficiently manage the simultaneous presence of constraints, model uncertainties, time-varying transmission delays and data-packet losses.

We will focus on a novel discrete time receding horizon strategy for NCSs, described by means of uncertain multi-model linear systems, under the occurrence of time-varying delays, data loss on the sensor-to-controller link and feedback command loss on the controller-to-actuator link. By resorting to a time-stamp protocol, data and feedback losses are separately accounted to make available a "usable" control move for the actuator logic within each sampling interval. The NCS stabilization problem will be dealt with a dual-mode predictive strategy. Off-line families of one-step con-

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trollable sets, "capable" to efficiently manage normal and data loss phases on actuator-sensor sides are first obtained by resorting to Independent-of-Delay (**IOD**) and Delay-Dependent (**DD**) stability concepts. Then at each sample time, an on-line receding horizon scheme is computed by deriving the smallest ellipsoidal set (**DD** or **IOD** type) compliant with the delay scenario.

The main merits of the proposed strategy can be summarized as follows: **1)** the computation of one-step controllable ellipsoids sequences capable to cope with time-delays occurrences; **2)** the computational resources (CPU power, memory resources and bandwidth requirements) are significantly modest when contrasted with competitor schemes.

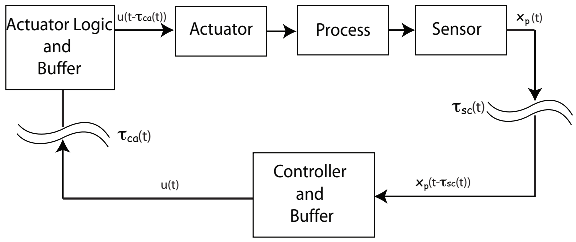

Fig. 1. Networked control system structure

2 Problem formulation

We will refer to the networked control scheme depicted in Fig. 1 where delay effects are taken into consideration from the sensor and actuator sides. Specifically:

Process - It is described by a multi-model uncertain discrete-time linear system

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t))u(t) \qquad (1)$$

where $t \in \mathbb{Z}_+ := \{0, 1, ...\}$, $x_p(t) \in \mathbb{R}^n$ denotes the state plant and $u(t) \in \mathbb{R}^m$ the control input. The parameter vector $\alpha(t) \in \mathbb{R}^l$ is assumed to lie in the unit simplex

$$\mathcal{P}_l := \left\{ \alpha \in \mathbb{R}^l : \sum_{i=1}^{l} \alpha_i = 1,\ \alpha_i \geq 0 \right\} \qquad (2)$$

and the system matrices $\Phi(\alpha)$ and $G(\alpha)$ belong to

$$\Sigma(\mathcal{P}_l) := \left\{ (\Phi(\alpha), G(\alpha)) = \sum_{i=1}^{l} \alpha_i (\Phi_i, G_i), \alpha \in \mathcal{P}_l \right\} \qquad (3)$$

the pairs (Φ_i, G_i) representing the polytope vertices $\Sigma(\mathcal{P}_l)$, viz. $(\Phi_i, G_i) \in \text{vert}\{\Sigma(\mathcal{P}_l)\}$, $\forall i \in \underline{l} := \{1, 2, \ldots, l\}$. Moreover, the control input is subject to the following saturation constraints

$$u(t) \in \mathcal{U},\ \forall t \geq 0,\ \mathcal{U} := \{u \in \mathbb{R}^m \,|\, u^T u \leq \bar{u}\}, \qquad (4)$$

with $\bar{u} > 0$ and $\mathcal{U}$ a compact subset of $\mathbb{R}^m$ containing the origin as an interior point.

Actuator and Controller buffers - The actuator buffer is in charge to memorize the last received command, hereafter denoted as u_{-1}^R, whereas the buffering unit on the controller side stores the last measurement received from the sensor-to-controller channel, named x_{-1}, and the last computed command u_{-1}^C.

Actuator logic - The actuator tracks data losses on the feedback channel: at each time instant t such a logic is instructed to apply the command $u(t)$ if available or conversely u_{-1}^R.

To properly treat data loss both on the plant-controller and controller-plant links, the sensor-to-controller and the controller-to-actuator cases need to be separately analyzed. We will suppose first that the delay on the command channel side $\tau_{ca}(t) : \mathbb{Z}_+ \to \mathbb{Z}_+$ is such that $\tau_{ca}(t) \leq \bar{\tau}_c,\ \forall t$, while the delay on the measurement side $\tau_{sc}(t) : \mathbb{Z}_+ \to \mathbb{Z}_+$ could be unbounded. On the contrary, the controller-to-actuator link (the feedback) is only subject to the actual induced delay $\tau_{ca}(t)$. Moreover, at each time instant t we shall define respectively with $\tau_m(t) \leq \tau_{ca}(t)$ and $\tau_c(t) \leq \tau_{sc}(t)$ the *age* of the state measurement used by the controller to compute the input and the *age* of the command used by the actuator. Then, on the plant-controller link at each time instant t when computing the input $u(t)$, the following age cumulative network latency should in principle be used: $\tau_{NL}(t) = \tau_m(t) + \tau_c(t - \tau_m(t))$. Since it is well-known that the round-trip delay $\tau_c(t - \tau_m(t))$ cannot be available at the controller side, the upper bound $\bar{\tau}_c$ on the controller-actuator link during both the controller design and the command computation $u(\cdot)$ phases is considered, i.e.

$$\tau(t) = \tau_m(t) + \bar{\tau}_c,\ \forall t. \qquad (5)$$

Hence, the following time-delay scenarios can occur on the communication channels:

- Sensor-to-Controller link
  - **Normal phase -** each time-delay occurrence is bounded, $\tau(t) < \bar{\tau}$, $\bar{\tau}$ being the maximum allowable transmission interval (MATI) (Walsh, Beldiman & Bushnell 2001);
  - **Data loss -** there exists $\bar{t}$ such that $\tau(\bar{t}) \geq \bar{\tau}$; the state measurement will be no longer available for the control action computation.
- Controller-to-Actuator link
  - **Normal phase -** at each time instant t the actuator receives a control action $u(t)$;
  - **Feedback loss -** there exists $\bar{t}$ such that $\tau_c(\bar{t}) \geq 1$, no control action is available for feedback.

Then, the problem statement is:

Network Constrained Stabilization (NCS) problem - Given the networked system depicted in Fig. 1 and the model plant (1)-(3), determine a state-feedback regulation strategy

$$u(\cdot) = g(x_p(\cdot)) \qquad (6)$$

complying with (4) such that, in the presence of time-delays (**Normal phases**) and packet dropouts (**Data losses**, **Feedback losses**) on both the communication channels, the regulated state trajectory is "jailed" inside the domain of attraction (DoA) due to (6) (Uniformly Ultimate Boundedness). □

In what follows we will be inspired by the class of computationally low demanding MPC schemes proposed e.g. in (Wan & Kothare 2003), (Angeli, Casavola, Franzè & Mosca 2008) which will be properly adapted to the NCS framework of Fig. 1. To this end, the following definition will be used (Blanchini & Miani 2008), (Raković, Kerrigan, Mayne & Lygeros 2006):

Definition 1 *Given the plant (1) and a controlled-invariant target set* $\mathcal{T}$, *the set of states i-step controllable to* $\mathcal{T}$ *is defined via the following recursion:*

$$\begin{aligned}\mathcal{T}_0 &:= \mathcal{T}\\ \mathcal{T}_i &:= \{x_p : \exists u \in \mathcal{U} : \Phi(\alpha)x_p + G(\alpha)u \in \mathcal{T}_{i-1}, \forall \alpha \in \mathcal{P}_l\}\end{aligned} \tag{7}$$

where $\mathcal{T}_i$ *is the set of states that can be steered into* $\mathcal{T}_{i-1}$ *using a single control move.*

Then, the structure of the proposed RHC algorithm is:

- *Off-line* - Two stabilizing state-feedback control laws (6) and the corresponding robust positively invariant regions (RPI) $\mathcal{T}_0^{DD}$ and $\mathcal{T}_0^{IOD}$ for (1)-(3) are first derived by resorting to **DD** and **IOD** stability concepts, respectively. Then, two sequences of N one-step ahead controllable sets $\{\mathcal{T}_i^{DD}\}$ and $\{\mathcal{T}_i^{IOD}\}$ are computed by enlarging $\mathcal{T}_0^{DD}$ and $\mathcal{T}_0^{IOD}$ under the requirement that each new state can be steered into $\mathcal{T}_0^{DD}$ (respectively $\mathcal{T}_0^{IOD}$) in a finite number of steps;
- *On-line* - At each time t and given the delayed state measurement from (1), the algorithm first determines the inner set $\mathcal{T}_i^{DD}$ (or $\mathcal{T}_i^{IOD}$) containing $x_p(t-\tau(t))$ with respect to the delay time instance (5). The current input $u(t)$ is finally computed by minimizing a running cost such that the one-step state prediction belongs to the successor set (**DD** or **IOD**), i.e.

  $$J(x_p(t-\tau(t)), u) := \max_{\forall \alpha \in \mathcal{P}_l} \|\Phi(\alpha)x_p(t-\tau(t)) + G(\alpha)u\|^2_{P_{i-1}}$$

  with $P_i = P_i^T > 0$ the shaping matrix of the one-step controllable set $\mathcal{T}_i$.

Finally, a time-stamping protocol equipped with a clock synchronization mechanism (Shaw 2001) must be used to properly identify the age of each data-packet sent from the sensor and controller nodes.

3 Off-line phase

In this section the conditions under which adequate control actions can be computed on the remote side and then correctly applied to the plant regardless of data loss and/or feedback loss events are formally stated.

3.1 DD/IOD robustly invariant terminal regions

Here, we derive the terminal regions according to the **DD** and **IOD** time-delay scenarios and the prescribed constraints (4). To this end, we consider a state-feedback control law

$$u(t) = K_{DD}\, x_p(t-\tau(t)) \tag{8}$$

which satisfies the **NCS problem** requirements for the regulated plant

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t))\, K_{DD}\, x_p(t-\tau(t)) \tag{9}$$

We will exploit a standard technicality in delayed systems (see (Fridman & Shaked 2005) and references therein). Then, the **DD** feedback control law (8) robustly stabilizes the plant and satisfies the prescribed constraints if the following matrix inequalities in the objective variables K_{DD}, P_{DD}, Q, R and $\tau_{max} \leq \bar{\tau}$, evaluated over the plant vertices (3), are satisfied

$$\begin{bmatrix} E^T P_{DD} E - \mathcal{S}_{DD} & 0 & \mathcal{A}_{DD}^{j^T} P_{DD} \\ 0 & \tau_{max}(R+Q) & \mathcal{B}_{DD}^{j^T} P_{DD} \\ P_{DD}\mathcal{A}_{DD}^j & P_{DD}\mathcal{B}_{DD}^j & P_{DD} \end{bmatrix} \geq 0,$$
$$j = 1, \ldots, l \tag{10}$$

$$\begin{bmatrix} \bar{u}^2 E^T P_{DD} E & \begin{bmatrix} K_{DD}^T \\ 0 \end{bmatrix} \\ \begin{bmatrix} K_{DD} & 0 \end{bmatrix} & I \end{bmatrix} \geq 0 \tag{11}$$

where $P_{DD} = P_{DD}^T \geq 0$, $R = R^T \geq 0$, $Q = Q^T \geq 0$ $E = \text{diag}\{I, 0\}$, $\mathcal{S}_{DD} \triangleq \text{diag}\{0, \tau_{max}\,(R+Q)\}$ and

$$\mathcal{A}_{DD}^j = \begin{bmatrix} I & I \\ \Phi_j - I + G_j K_{DD} & -I \end{bmatrix},\ \mathcal{B}_{DD}^j = \begin{bmatrix} 0 \\ -G_j K_{DD} \end{bmatrix},$$
$$j = 1, \ldots, l.$$

As regards as the **IOD** case, we search for a pair (P_{IOD}, K_{IOD}) complying with $\tau(t) \leq \bar{\tau}$. To this end, the constrained **IOD** feedback control law

$$u(t) = K_{IOD}\, x_p(t-\tau(t)) \tag{12}$$

stabilizes the plant

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t))K_{IOD}x_p(t-\tau(t)) \tag{13}$$

if

$$\begin{bmatrix} E^T P_{IOD} E - \mathcal{S}_{IOD} & \mathcal{A}_{IOD}^{j^T} P_{IOD} \\ P_{IOD}\mathcal{A}_{IOD}^j & P_{IOD} \end{bmatrix} \geq 0,\ j = 1, \ldots, l \tag{14}$$

$$\begin{bmatrix} \bar{u}^2 E^T P_{IOD} E & \begin{bmatrix} K_{IOD}^T \\ 0 \end{bmatrix} \\ \begin{bmatrix} K_{IOD} & 0 \end{bmatrix} & I \end{bmatrix} \geq 0 \tag{15}$$

where

$$\mathcal{S}_{IOD} \triangleq \text{diag}\{S, 0\}, \; S = S^T \geq 0,$$

$$\mathcal{A}_{IOD}^{j} = \begin{bmatrix} I & I \\ (\Phi_j - I + G_j K_{IOD}) & -I \end{bmatrix}, j = 1, \ldots, l.$$

Hence, the ellipsoidal sets

$$\mathcal{E}_{DD} := \{x_p \in \mathbb{R}^n | x_p^T Q_{DD} x_p \leq 1\}, Q_{DD} = Q_{DD}^T > 0,$$

$$\mathcal{E}_{IOD} := \{x_p \in \mathbb{R}^n | x_p^T Q_{IOD} x_p \leq 1\}, Q_{IOD} = Q_{IOD}^T > 0,$$

arising from the inequalities (10), (11) and (14), (15) are robust positively invariant regions for the closed-loop state evolutions (9), (13) complying with the input constraints (4), viz. $K_{DD}\mathcal{E}_{DD} \subset \mathcal{U}$, $K_{IOD}\mathcal{E}_{IOD} \subset \mathcal{U}$.

3.2 One-step ahead Ellipsoidal controllable sets

The aim is to characterize all the states one-step controllable to a given target set $\mathcal{T}$. To extend such a concept to the proposed framework, it is important to notice that the one-step state prediction needs to be evaluated along the model

$$x(t+1) = \Phi(\alpha(t))x(t) + G(\alpha(t))u(t) \quad (16)$$

with $u(t)$ chosen according to a delayed state plant information. The rationale behind the introduction of (16) relies on the necessity to properly take into account an unavoidable time misalignment existing between the measured plant state which is sent to the controller and the delayed state exploited instead by the controller unit to compute the command input to be sent to the plant (see Fig. 1). To better clarify, let us take a look to Fig. 2.

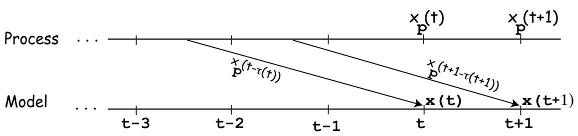

Fig. 2. Process/model discrepancy

There, at the generic time instant t, the model (16) has knowledge on the state measurement generated $\tau(t)$ instants before, i.e. $x(t) = x_p(t - \tau(t))$, while the process state is $x_p(t)$. Since in general $x(t) \neq x_p(t)$, there exists a discrepancy between the process (1) and the model (16) that, if not properly treated, can lead to erroneous input evaluations. There is in fact no guarantee that if $x(t) \in \mathcal{T}_i$ the same holds true for $x_p(t)$. A possible way to comply with the above reasoning is to determine the sequence of sets $\mathcal{T}_i$ by resorting to the following one-step transition map, valid for $0 \leq \tau(t) \leq \bar{\tau}$

$$x(t+1) = \Phi(\alpha(t))x(t - \tau(t)) + G(\alpha(t))u(t) \quad (17)$$

Note that the delayed state $x(t - \tau(t))$ in (17) is instrumental to take care, at each instant t, the difference between the state measurement $x(t)$ and the *real* plant state $x_p(t)$. By virtue of (17), if $x(t) \in \mathcal{T}_i$ with $x(t) \neq x_p(t)$, the same holds for $x_p(t)$, and there exists a command $u(t)$ that drives $x(t+1)$ into $\mathcal{T}_{i-1}$ for all $\tau(t) \in [0, \; \bar{\tau}]$. To recast such an idea into a computable scheme, explicit time-delay dependencies in the auxiliary model (17) need to be derived. By considering the auxiliary state $y(t)$, the following descriptor form results

$$\begin{bmatrix} x(t+1) \\ 0 \end{bmatrix} = \begin{bmatrix} y(t) + x(t) \\ -y(t) + \Phi(\alpha(t))x(t-\tau(t)) \\ +G(\alpha(t))u(t) - x(t) \end{bmatrix} \quad (18)$$

Hence, by noticing that $x(t-\tau(t)) = x(t) - \sum_{j=t-\tau(t)}^{t-1} y(j)$ and by imposing the worst-case occurrence on the time-varying delay $\tau(t)$, i.e. $\tau(t) = \bar{\tau}, \; \forall t$, we have that

$$\bar{E}\bar{x}(t+1) = \bar{\Phi}(\alpha(t))\bar{x}(t) + \bar{G}(\alpha)u(t) - \bar{G}_y(\alpha(t)) \sum_{j=t-\bar{\tau}}^{t-1} y(j) \quad (19)$$

where $\bar{x}(t) = [x(t)^T \; y(t)^T]^T$, $\bar{E} = \text{diag}\{I, 0\}$, $\bar{\Phi}(\alpha(t)) = \begin{bmatrix} I & I \\ \Phi(\alpha(t)) - I & -I \end{bmatrix}$, $\bar{G}(\alpha(t)) = \begin{bmatrix} 0 \\ G(\alpha(t)) \end{bmatrix}$ and $\bar{G}_y(\alpha(t)) = \begin{bmatrix} 0 \\ \Phi(\alpha(t)) \end{bmatrix}$.

The computation of set recursions (7) along the system dynamics (19) prescribes that all the auxiliary variables $y(j) = x(j+1) - x(j)$, $j = t - \bar{\tau}, \ldots, t-1$, need to be included in the augmented system. Unfortunately the obtained state space description has huge dimension and could be computationally intractable. The following proposition provides inner approximations of the exact one-step controllable sets related to the description (17).

Proposition 1 *Let $\mathcal{T}_0 \neq \emptyset$ be a given robustly invariant ellipsoidal region complying with the input constraints and $x_{aug} = \begin{bmatrix} x^T \; y^T \; z_1^T \; z_2^T \end{bmatrix}^T$ the augmented state describing the dynamics (19) with $z_1, z_2 \in \mathbb{R}^n$ accounting for all the cumulative sum vectors $y(j)$, $j = t - \bar{\tau}, \ldots, t-1$. Then, the ellipsoidal set sequence*

$$\begin{aligned} \mathcal{E}_0 &= \mathcal{T}_0 \\ \mathcal{E}_i &= Proj_x\{In[\bar{x}_{aug} \in \mathbb{R}^{4n} \text{ with } z_1, z_2 \in \mathcal{E}_{i-1} : \exists u \in \mathcal{U} : \\ &\quad Proj_x\{\bar{\Phi}_{aug}(\alpha)\bar{x}_{aug} + \bar{G}_{aug}(\alpha)u\} \in \mathcal{E}_{i-1}], \forall \alpha \in \mathcal{P}_l\} \end{aligned} \quad (20)$$

$$\bar{\Phi}_{aug}(\alpha) = \begin{bmatrix} I & I & 0 & 0 \\ \Phi(\alpha) - I & -I & -\bar{\tau}\Phi(\alpha) & \bar{\tau}\Phi(\alpha) \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & \Phi(\alpha) \end{bmatrix},$$

$$\bar{G}(\alpha)_{aug} = \begin{bmatrix} 0 & G^T(\alpha) & 0 & 0 \end{bmatrix}^T$$

if non-empty, satisfies $\mathcal{E}_i \subset \mathcal{T}_i$, where $In[\cdot]$ denotes the inner ellipsoidal approximation operator.

Proof - Assume that the sequence of one-step controllable sets $\mathcal{T}_i$ for the descriptor form (19) has been computed by resorting to the whole augmented state description $\left[x^T(t)\, y^T(t)\, y^T(t-1)\; \ldots\; y^T(t-\bar{\tau})\right]^T$. Note that, because the auxiliary variables $y(\cdot)$ are linear combinations of $x(\cdot)$, at each recursion all the initial vectors $x(j),\ j = t-\bar{\tau}, \ldots, t-1$ must lie in $\mathcal{T}_i$ in order to ensure that the one-step evolution of (17) belongs to $\mathcal{T}_{i-1}$. In view of this reasoning, a way to deal with the one-step controllable sets computation is to impose that at each recursion $x(j) \in \mathcal{T}_{i-1},\ j = t-\bar{\tau}, \ldots, t-1$. The latter is admissible thanking to the nesting property of the one-step controllable set sequence, see (Blanchini & Miani 2008).

Now, each sample of the initial segments $x(\cdot)$ belongs to the same set $\mathcal{T}_{i-1}$, and a simple way to proceed is to consider two vectors, namely $z_1,\ z_2 \in \mathbb{R}^n$, characterizing the terms in the one-step differences $y(\cdot)$. Specifically z_2 denotes a slack variabile representing the first element in the one-step difference whereas z_1 is a term characterizing all the time delayed states with respect to z_2 along all the possible initial segments. Therefore the computed one-step controllable set, say $\mathcal{E}_i$, is an inner approximation of the exact set $\mathcal{T}_i$ and the descriptor form (19) consequently becomes:

$$\bar{E}_{aug}\, \bar{x}_{aug}(t+1) = \bar{\Phi}_{aug}(\alpha(t))\, \bar{x}_{aug}(t) + \bar{G}_{aug}(\alpha(t))u(t)$$

with $\bar{x}_{aug}(t) = \left[x^T(t)\, y^T(t)\, z_1^T(t)\, z_2^T(t)\right]^T$, $\bar{E}_{aug} = \text{diag}\{I, 0, I, I\}$ and the recursions (20) result. □

A computable procedure for the sets $\mathcal{E}_i$ is provided in (Famularo, Franzè & Tedesco 2012).

3.3 Off-line time-delays and data loss management

Here, we will discuss the time-delay configurations under which the networked system can operate.

3.3.1 Sensor-to-Controller and Controller-to-Actuator normal phases

In such a case the time-delay can be managed by computing two nested one-step controllable ellipsoidal sequences with $N+1$ elements ($N > 0$) $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$, where both the sequences are obtained by using the inner approximation (20), such that

$$\mathcal{T}_0^{DD} \subseteq \mathcal{T}_N^{IOD} \tag{21}$$

The key idea can be stated as follows: the ellipsoidal sequences are achieved on the hypothesis that the time-delay occurrence is $\tau(t) \leq \tau_{max}$ for $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\tau_{max} < \tau(t) \leq \bar{\tau}$ for $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$. Now, at each time instant and on the basis of the information $\tau(t)$, if the current measurement $x_p(t-\tau(t))$ belongs to $\mathcal{T}_i^{DD}$ $\left(\text{resp. } \mathcal{T}_i^{IOD}\right)$, there exists a command u_i, compatible with (4) and capable to drive the state to $\mathcal{T}_{i-1}^{DD}$ $\left(\text{resp. } \mathcal{T}_{i-1}^{IOD}\right)$. Therefore, there exists an admissible control strategy which steers in a finite number of steps any initial state $x(0) \in \mathcal{T}_N^{DD}$ $\left(\text{resp. } \mathcal{T}_N^{IOD}\right)$ to the terminal (target) set $\mathcal{T}_0^{DD}$ $\left(\text{resp. } \mathcal{T}_0^{IOD}\right)$.

3.3.2 Data loss

Let us define a data loss event:

Definition 2 *Let $\Delta^{DL} := [\bar{t}_{in},\ \bar{t}_{fin}]$ be a time interval. A data loss occurs if $\tau(t) > \bar{\tau},\ \forall t \in \Delta^{DL}$.*

Under data losses, the combined use of the sequences $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$ is not able to deal with all the time-varying delay occurrences. In fact, it may happen that when the state lies in any ellipsoid of the sequences $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$, there is no guarantee on the length of Δ^{DL}. Then, two questions arise: **1)** Which is the maximum admissible length of Δ^{DL}? **2)** How is it possible to proceed when $x(\cdot) \in \mathcal{T}_0^{DD}$ or $\mathcal{T}_0^{IOD}$?

The question 1) is related to the strict connection of the command $u(t)$ to the construction algorithm of the sequences $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ thanking to the state measurement availability. By recalling from *Proposition 1* that such ellipsoidal sequences are generated by taking into account the process/model discrepancy (see Fig. 2) due to time-delay occurrences within $[0,\ \bar{\tau}]$, then the command computed at time t can be used in an open-loop fashion if Δ^{DL} lasts less than $\bar{\tau}$.

As regards the second question, the technical obstacle is avoided by imposing that the **DD** and **IOD** sequences are computed by satisfying the condition indicated in the following statement.

Statement 1 *Let $x^+ := \Phi(\alpha)\, x,\ \forall \alpha \in \mathcal{P}, \forall x \in \mathcal{T}_0^{DD}$ and $\forall x \in \mathcal{T}_0^{IOD}$ be the one-step state evolution under zero-input $u \equiv 0_m$, then*

$$x^+ \subseteq \mathcal{T}_N^{DD} \cup \mathcal{T}_N^{IOD} \tag{22}$$

If (22) holds, one has that the zero input $u \equiv 0_m$ can be applied in place of K_{DD} when no state measurements are available. In what follows, we shall denote as $\mathcal{T}_{sup}$ the ellipsoidal region complying with (22).

3.3.3 Feedback loss

The following definition is considered:

Definition 3 *Let $\Delta^{FL} := [\bar{t}_{in},\ \bar{t}_{fin}]$ be a time interval such that $\bar{t}_{fin} - \bar{t}_{in} \leq \bar{\tau}_c$. A feedback loss occurs if $\tau_c(t) \geq 1,\ \forall t \in \Delta^{FL}$.*

In this case even if the actuator does not receive new packets, an admissible command input must be applied to the plant $\forall t \in \Delta^{FL}$. To comply with this situation it is mandatory to guarantee that a new computed command $u(t)$ is available after $\bar{t}_{fin} - \bar{t}_{in}$ time instants before than u_{-1}^R becomes no longer admissible. Then the consequence is that the following conditions on the controller-to-actuator channel must be imposed

$$\bar{\tau}_c \leq \tau_{max} \tag{23}$$

$$2\,\tau_{max} \leq \bar{\tau} \tag{24}$$

The necessity of (23) comes from the argument that since within the time interval Δ^{FL} the plant operates in open-loop and the actuator applies the last received input u^R_{-1}, the length of consecutive feedback loss events is at most τ_{max}. The requirement (24) while ensures that the command $u(t)$ computed with respect to the upper bound $\bar{\tau}$ is usable for τ_{max} time instants.

4 On-line phase

In what follows, we will resort to a time-stamp mechanism in order to characterize the data-packet dispatch along the sensor-to-controller and controller-to-actuator channels. Specifically the event (x_p, t_x) denotes the state measurement x_p sent at the time instant t_x, whereas (u, t_u) has the following meaning: u is the command sent by the controller and marked by the stamp t_u. On the controller side, the computation of the control action $u(t)$ is performed by using the following arguments. At each instant t the logic on the controller side first considers the most recent data-packet (x_p, t_x) and selects the set containing x_p. Then two scenarios could arise:

a) $\tau(t) \leq \bar{\tau}$: if $x_p \in \mathcal{T}_i^{IOD}$ the set $\mathcal{T}_i^{IOD}$ is selected, otherwise determine the smallest index i such that $x_p \in \mathcal{T}_i^{DD}$;

b) $\tau(t) > \bar{\tau}$: the state x_p is not available for checking its membership to **IOD** or to **DD** set sequences.

Then, an admissible input $u(t)$ is computed by minimizing the performance index

$$J_{i(t)}(x_p, u) = \max_j \|\Phi_j x_p + G_j u\|^2_{P_{i(t)-1}} \qquad (25)$$

as below detailed:

- If **a)** holds true and $x_p \in \mathcal{T}_i^{IOD}$(or $\mathcal{T}_i^{DD}$) then

$$u(t) = \arg\min J_{i(t)}(x_p, u) \text{ s.t.} \qquad (26)$$

$$\Phi_j x_p + G_j u \in \mathcal{T}^{IOD}_{i(t)-1} (\text{or } \mathcal{T}^{DD}_{i(t)-1}), u \in \mathcal{U}, j = 1, \ldots, l. \qquad (27)$$

- The case **b)** envisages the following events:

(1) if $x_{-1} \in \mathcal{T}^{DD}_{i+1}$ or $x_{-1} \in \mathcal{T}^{IOD}_{i+1}$ then $u(t) = u^C_{-1}$;

(2) if $x_{-1} \in \mathcal{T}^{DD}_0$ or $x_{-1} \in \mathcal{T}^{IOD}_0$ then solve (26)-(27) with x_{-1} in place of x_p and $\mathcal{T}_{sup}$ in place of $\mathcal{T}_i^{IOD}$.

Finally, by referring to the scheme of Fig. 1 the actuator logic implements the following strategy:

$$u(t - \tau_c(t)) = \begin{cases} u, \text{ if } (u, t_u) \text{ is available} \\ u^R_{-1}, \text{ if a feedback loss occurs} \end{cases} \qquad (28)$$

Then, the above developments allow to write down a computable MPC scheme.

NCS-MPC-Algorithm

Initialization

0.1 Given the scalars $\bar{\tau}_c$, τ_{max} *and* $\bar{\tau}$ *satisfying the requirements (23)-(24), compute the nonempty robust invariant ellipsoidal regions* $\mathcal{T}_0^{DD} \subset \mathbb{R}^n$, $\mathcal{T}_0^{IOD} \subset \mathbb{R}^n$ *and the stabilizing state feedback gains* K_{DD}, K_{IOD} *complying with the prescriptions of Section 3.1;*

0.2 Generate the sequences of N *one-step controllable sets* $\mathcal{T}_i^{DD}$ *and* $\mathcal{T}_i^{IOD}$ *complying with (21) and (22). Compute* $\mathcal{T}_{sup}$;

0.3 Store the ellipsoids $\{\mathcal{T}_i^{DD}\}_{i=0}^N$, $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$ *and* $\mathcal{T}_{sup}$.

On-line phase

SENSOR SIDE

for all $t \in \mathbb{Z}_+$

1.1 send the packet (x_p, t_{x_p}) *with* $t_{x_p} = t$ *the time-stamp;*

CONTROLLER SIDE

for all $t \in \mathbb{Z}_+$

1.1 If a packet (x_p, t_{x_p}) *is arrived then*

1.1.1 compute $\tau(t) = \tau_m(t) + \bar{\tau}_c$ *with* $\tau_m(t) = t - t_{x_p}$;

1.1.2 If $\tau(t) \geq \bar{\tau}$ *goto 1.2.1,*

1.1.3 otherwise

1.1.3.1 If there exists $i(t) := \min\{i : x_p \in \mathcal{T}_i^{IOD}\}$ *then*

a- If $i(t) = 0$ *then* $u(t) = K_{IOD} x_p$

b- else solve (26)-(27);

1.1.3.2 else find $i(t) := \min\{i : x_p \in \mathcal{T}_i^{DD}\}$

a- If $i(t) = 0$ *then* $u(t) = K_{DD} x_p$

b- else solve (26)-(27);

1.1.3.3 If $t_{x_p} > t_{u^C_{-1}}$ *then* $x_{-1} := x_p$ *and* $u^C_{-1} := u(t)$;

1.2 If a data loss occurs (no packed arrived) then

1.2.1 if $x_{-1} \in \mathcal{T}_0^{DD}$ *or* $x_{-1} \in \mathcal{T}_0^{IOD}$

1.2.1.1 solve (26)-(27) with x_{-1} *in place of* x_p *and* $\mathcal{T}_{sup}$ *in place of* $\mathcal{T}_i^{IOD}$;

1.2.1.2 update $u^C_{-1} := u(t)$;

1.2.2 else $u(t) = u^C_{-1}$;

1.3 Send the packet (u, t_u).

ACTUATOR SIDE

for all $t \in \mathbb{Z}_+$

1.1 If a packet (u, t_u) *is arrived*

1.1.1 apply u.

1.1.2 If $t_u > t_{u^R_{-1}}$ *then* $u^R_{-1} := u$ *and* $t_{u^R_{-1}} := t_u$;

1.2 else apply u^R_{-1};

One of the crucial points is to prove the feasibility retention and closed-loop stability of the scheme.

Proposition 2 *Let the sequences of sets* $\mathcal{T}_i^{DD}$ *and* $\mathcal{T}_i^{IOD}$ *be non-empty and* $x_p(0) \in \mathcal{T}_N^{DD} \cup \mathcal{T}_N^{IOD}$. *Then, the* **MPC-NCS-Algorithm** *always satisfies the constraints and ensures Uniformly Ultimate Boundedness for all* $\alpha \in \mathcal{P}_l$ *and for all time-delay occurrences* $\tau_c(t) \leq \bar{\tau}_c$ *and* $\tau_m(t)$.

Proof - Let us consider without loss of generality that at the generic instant t the state $x_p \in \mathcal{T}_{i(t)}^{DD}$.
First note that by construction there exists an input vector u satisfying the input constraints (4) such that the one-step state evolution $\Phi(\alpha)x_p + G(\alpha)u$ belongs to $\mathcal{T}_{i(t)-1}^{DD}$, $\forall \alpha \in \mathcal{P}_l$ and $\forall \tau(t)$ with $\tau_c(t) \leq \tau_{max}$ and $\tau_m(t)$ unbounded. Then, thanks to the recursion (7) and under the constraints (21) and (22) at the next time instant $t+1$, the existence of a solution $u(t+1)$ for the steps 1.1.3.1, 1.1.3.2 and 1.2.1.1 is ensured. Under a data loss occurrence, thanking to (23) the stored command u_{-1}^C, though not optimal, is admissible at time $t+1$ and, because the number of consecutive data losses is less than τ_{max}, feasibility is retained.
On the actuator side the feasibility is guaranteed thanking to the properties of the ellipsoidal sequences $\mathcal{T}_i^{DD}$ and $\mathcal{T}_i^{IOD}$ and to the communication upper bound requirements (23) and (24). In fact, at each time instant the actuator logic is able to apply an admissible control input because the last stored command u_{-1}^R is updated at most after τ_{max} time instants.
Finally, Uniformly Ultimate Boundedness follows by exploiting the same arguments: under a *Normal phases* regime each initial state $x_p(0) \in \mathcal{T}_N^{DD} \cup \mathcal{T}_N^{IOD}$ is by construction driven to the terminal sets; under data loss or feedback loss events the trajectory is in the worst case confined to $\mathcal{T}_N^{DD} \cup \mathcal{T}_N^{IOD}$ thanking to (21) and (22). □

5 Illustrative example

The aim of this example is to present results on the effectiveness of the proposed MPC strategy. Numerical comparisons with the MPC scheme proposed by Pin and Parisini (Pin & Parisini 2011), hereafter denoted as **MPC-NDC-Algorithm** will be considered both in terms of control performance and memory resources.
A path-tracking problem for a mobile autonomous robot is here considered (see Fig. 3). A state-space model and numerical values of the physical parameters can be found in (Torby 1984). The plant has been linearized around a given nominal solution $(\hat{x}_p(t),\ \hat{u}(t))$, and by defining $\tilde{x}_p(t) := x_p(t) - \hat{x}_p(t)$, $\tilde{u}(t) := u(t) - \hat{u}(t)$, with $x_p(t) = [x(t),\ y(t),\ v(t),\ \theta(t),\ \omega(t)]^T$ and $u(t) = [T_L(t),\ T_R(t)]^T$, a robust Polytopic Linear Differential Inclusion (PLDI) is derived as detailed in (Franzè, Famularo & Tedesco 2014). Then, a discrete-time model description is achieved by using the sampling time $T_c = 0.01\,sec.$ and componentwise constraints on the input torque are prescribed: $-0.5\,Nm \leq \tilde{u}_i < 0.5\,Nm, i = 1,2$. Finally, data are supposed to be exchanged using a non-acknowledged communication protocol (UDP), the maximum round trip delay is approximately 20 (the MATI value $\bar{\tau}$). The network latency on the controller side $\tau(t)$ randomly varies within $\bar{\tau}$ except for the interval [40, 90] *sec* where $\tau(t) \in [0,\ 30]$. On the other hand, the latency on the actuator side is such that $\tau_c(t) < 1$ expect for the interval [40, 90] *sec* where it goes randomly within $[0, 10]$.

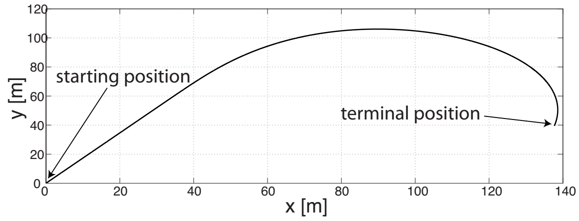

Fig. 3. Robot path

For the proposed method, by iteratively solving (10)-(11), the upper-bound τ_{max} has been approximated to $100\,ms$. Then, two families $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$ of $N = 50$ ellipsoids have been computed under the satisfaction of conditions (21) and (22) with $\mathcal{T}_{sup} \subseteq \mathcal{T}_{11}^{DD}$. In order to implement The **MPC-NDC-Algorithm** has been implemented according to: controller sequence length: $N_c = 33$; stage cost: $h(\tilde{x}_p) = \tilde{x}_p^T Q \tilde{x}_p + \tilde{u}^T R \tilde{u}$ $(Q = I_n,\ R = I_m)$. The initial state has been set to $\tilde{x}_p(0) = [0.5, 0.3, 0, \frac{\pi}{3}, 0]^T$. In Fig. 4 the regulated state evolutions with respect to the path to be tracked (Fig. 3) under time-delay occurrences are depicted.
During the first $40\,sec.$ the communication time-delays are such that $\tau(t) \leq \bar{\tau}$ and $\tau_c(t) < 1$, therefore the proposed strategy and the **MPC-NDC** algorithm apply less conservative control actions so achieving shorter settling times, see Figs. 4 and 5.
When data loss and feedback loss scenarios are concerned ($[40, 90]\,sec.$), note that the controller-to-sensor delay $\tau_c(t)$ goes randomly beyond the sensor-to-controller latency MATI $\bar{\tau}$. For the **NCS-MPC** scheme, this implies that at the controller side $u(t)$ is computed so that $x^+ \subseteq \mathcal{T}_{sup}$ (**Step 1.2.1.1**) or assigned to u_{-1}^C (**Step 1.2.1.2**). Note that at $t = 55\,sec.$ the application of $u(t)$ (computed by means of the **Step 1.2.1.1)**) leads to a "jump" into $\mathcal{T}_8^{DD} \subseteq \mathcal{T}_{sup}$ of the state x^+ and a new input sequence is computed by following the prescriptions of the **NCS-MPC** algorithm as shown in Fig. 5. During such a phase, the regulated robot attitude initially departs from the equilibrium condition because the packet dropouts impose the *worst* control action on the mobile robot, see the grey zone in Fig. 4. Notice that, by considering the control performance achieved by the **MPC-NDC** scheme, even if the regulated state evolutions do not suffer of abrupt divergences from the nominal path, they show a slow rate of convergence to the desired trajectory. As a conclusion, it appears that the two strategies perform quite similarly in all the time-delay scenarios. Finally, by comparing the amount of needed memory resources we can note that the actuator buffer length of the **MPC-NDC** strategy requires higher memory resources (an input sequence of length $N_c = 33$ must be stored) than to that pertaining to the the proposed scheme (a single command u_{-1}^R). As a consequence, since at each time instant an entire sequence of N_c control moves must be sent from the controller to the actuator, the use of the **MPC-NDC** algorithm needs a larger bandwidth of the controller-to-sensor

channel. This appears evident by looking at the needed average CPU times (sec/step) and the average information transmitted (bit/step) from the controller to the actuator if 32bit are needed to encode a scalar, respectively: 0.0061 for **MPC-NDC** and 0.0002 for **NCS-MPC**; 1024 for **MPC-NDC** and 96 for **NCS-MPC**.

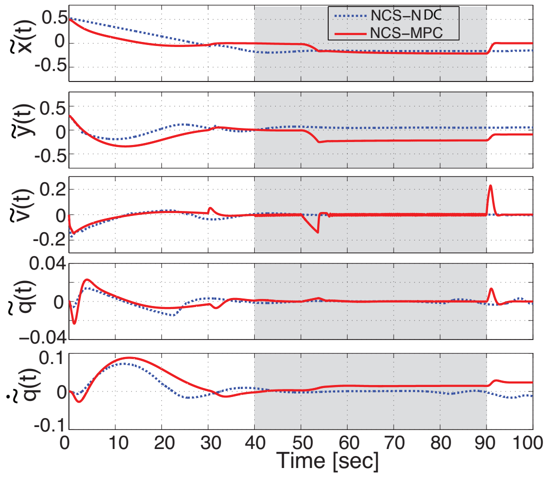

Fig. 4. Regulated state evolutions

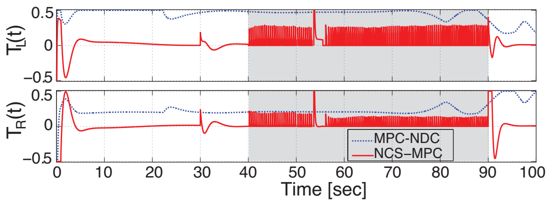

Fig. 5. Command inputs

6 Conclusions

In this paper, a novel receding horizon strategy for uncertain networked systems subject to input saturations and data losses has been presented. The key idea was to develop a worst-case framework based on set-invariance concepts and ellipsoidal calculus to manage the absence on state measurements due to large network delays and to deal with unreliable communication channels without affecting closed-loop stability and constraints satisfaction. Finally, an illustrative example testifies that the proposed strategy allows to hold the control performance at a satisfactory level in view of unrecoverable packet dropout events at the expense of modest computational and resources requirements.

References

Angeli, D., Casavola, A., Franzè, G. & Mosca, E. (2008), 'An ellipsoidal off-line mpc scheme for uncertain polytopic discrete-time systems', *Automatica* **44**, 3113–3119.

Blanchini, F. & Miani, S. (2008), *Set-Theoretic Methods in Control*, Birkäuser.

Cloosterman, M. B. G., van de Wouw, N., Heemels, W. P. M. H. & Nijmeijer, H. (2009), 'Stability of networked control systems with uncertain time-varying delays', *IEEE Trans. Auto. Contr.* **54**.

Famularo, D., Franzè, G. & Tedesco, F. (2012), Controllability analysis of uncertain polytopic systems with time-varying state delay, *in* '51th IEEE Conference on Decision and Control', pp. 2352–2357.

Franzè, G., Famularo, D. & Tedesco, F. (2011), Receding horizon control for constrained networked systems subject to data-losses, *in* '50th IEEE Conference on Decision and Control and European Control Conference', IEEE, pp. 5260–5265.

Franzè, G., Famularo, D. & Tedesco, F. (2014), Constrained predictive strategies for networked systems subject to time-varying delays, Technical Report DIMES14-06, DIMES - Università degli Studi della Calabria, http://tedescof.files.wordpress.com/2014/03/rep.pdf.

Fridman, E. & Shaked, U. (2005), 'Delay-dependent $h-infinity$ control of uncertain discrete delay systems', *European Journal of Control* **11**, 29–37.

Heemels, W. P. M. H., Teel, A. R., van de Wouw, N. & Nesic, D. (2010), 'Networked control systems with communication constraints: tradeoffs between transmission intervals, delays and performance', *IEEE Trans. Auto. Contr.* **55**.

Hespanha, J., Naghshtabrizi, P. & Xu, Y. (2007), 'A survey of recent results in networked control systems', *Proc. of the IEEE* **95**, 138–162.

Kurzhanski, A. & Valyi, I. (1997), *Ellipsoidal Calculus*, Birkhäuser.

Montestruque, L. A. & Antsaklis, P. J. (2003), 'On the model-based control of networked systems', *Automatica* **39**, 1837–1843.

Montestruque, L. A. & Antsaklis, P. J. (2004), 'Stability of model-based networked control systems with time-varying transmission times', *IEEE Trans. Auto. Contr.* **49**.

Muñoz de la Peña, D. & Christofides, P. D. (2008), 'Lyapunov-based model predictive control of nonlinear systems subject to data losses', *IEEE Trans. Auto. Contr.* (53), 2076–2089.

Nillson, J. (1998), Real-time Control Systems with Delays, PhD thesis, Lund Institute of Technology.

Pin, G. & Parisini, T. (2011), 'Networked predictive control of uncertain constrained nonlinear systems: Recursive feasibility and input-to-state stability analysis', *IEEE Trans. Auto. Contr.* **56**, 72–87.

Quevedo, D. E. & Gupta., V. (2013), 'Sequence-based anytime control', *IEEE Trans. Auto. Contr.* **58**.

Quevedo, D., Silva, E. & Goodwin, G. (2007), Packetized predictive control over erasure channels., *in* 'Proc. 2007 American Control Conference', pp. 1003–1008.

Raković, S. V., Kerrigan, E. C., Mayne, D. Q. & Lygeros, J. (2006), 'Reachability analysis of discrete-time systems with disturbances', *IEEE Trans. Auto. Contr.* **51**.

Shaw, A. C. (2001), *Real-time systems and software*, Wiley.

Torby, B. (1984), *Energy Methods. Advanced Dynamics for Engineers*, Mechanical Engineer, Holt, Rinehart, and Winston.

Walsh, G., Beldiman, O. & Bushnell, L. (2001), 'Asymptotic behavior of nonlinear networked control systems', *IEEE Trans. Auto. Contr.* **46**, 1093–1097.

Wan, Z. & Kothare, M. V. (2003), 'An efficient off-line formulation of robust model predictive control using linear matrix inequalities', *Automatica* **39**, 837–846.

Fig. 6:

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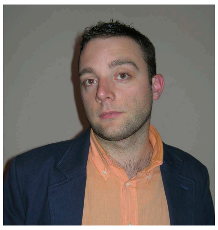

Fig. 7:

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Fig. 8:

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