Physics of Fluids

Physically based formula for the maximum scour depth induced by a propeller jet

Cite as: Phys. Fluids **35**, 035113 (2023); https://doi.org/10.1063/5.0140666
Submitted: 29 December 2022 • Accepted: 10 February 2023 • Published Online: 07 March 2023

Francesco Coscarella, Giuseppe Curulli, Nadia Penna, et al.

AIP Publishing

Physics of Fluids ARTICLE scitation.org/journal/phf

Physically based formula for the maximum scour depth induced by a propeller jet

Cite as: Phys. Fluids **35**, 035113 (2023); doi: 10.1063/5.0140666
Submitted: 29 December 2022 · Accepted: 10 February 2023 ·
Published Online: 7 March 2023

View Online Export Citation CrossMark

Francesco Coscarella,[a] Giuseppe Curulli, Nadia Penna, and Roberto Gaudio

AFFILIATIONS

Dipartimento di Ingegneria Civile, Università della Calabria, 87036 Rende, CS, Italy.

[a]Author to whom correspondence should be addressed: francesco.coscarella@unical.it

ABSTRACT

In the last two decades, many studies have been dedicated to the employment of predictive formulas able to assess the maximum scour depth produced by propeller jets. Owing to the complexity of the phenomenon, most of the literature formulas were built on empirical arguments, making them susceptible to scale issues and not fully coherent with the physics underpinning the scouring problem. Recent studies exploited the phenomenological theory of turbulence and the paradigms of sediment incipient-motion theory to derive a predictive formula of the maximum equilibrium scour depth in different cases: scour produced by jet, scour at bridge piers, and scour downstream of hydrokinetic turbines. In the present study, using the same considerations of the aforementioned works, we propose a new model that allows the derivation of a predictive physically based formula that includes all the relevant parameters controlling the scouring process induced by the propeller rotation. The theory is validated at the laboratory scale with experimental published data: a good agreement between theoretical predictions and literature data is shown. The proposed design formula clearly indicates that, when tested against experimental data, it leads to lower scattering levels than those obtained with empirical literature formulas.

I. INTRODUCTION

The conventional ship propulsion system is composed of a rotary motor turning a propeller, generally having more than three blades.[1] The system generates a turbulent jet that impacts the sea bottom up to various propeller diameters from it.[2] This effect assumes engineering interest in shallow water, such as in the harbor basins and navigation channels. As highlighted by many authors,[3–5] the turbulent propeller jet flow can produce a secondary small scour hole directly underneath the propeller, a main scour hole immediately downstream of this latter, and a deposition mound farther (Fig. 1).

The localized scour is caused by the shear stresses, triggered by the vortices, which surpass the critical bed shear stress for the inception of sediment motion. As the scour hole develops, the erosive intensity reduces, until the so-called equilibrium state is achieved.[6] Calculating the maximum scour depth produced under these conditions is paramount for a wide range of engineering applications, including the design and maintenance of harbor structures.[7]

Following Hamill *et al.*,[2] for the simplest case of one propeller, uniform sediment, and unconfined conditions (i.e., in the absence of structures in front of the propeller face), considering the maximum scour depth at the equilibrium state, d_s, one can write

$$d_s = f(U_0, D_p, d_{50}, h_0, h, \rho, \rho_s, g, \nu), \tag{1}$$

where U_0 is the efflux velocity, D_p is the external diameter of the propeller, d_{50} is the median diameter of bed sediments, h_0 is the vertical distance between the propeller axis and the initial bed level (i.e., the propeller gap), h is the water depth, ρ is the water density, ρ_s is the sediment density, g is the gravity acceleration, and ν is the kinematic viscosity of water.

Alternatively, in dimensionless form, the maximum equilibrium scour depth, normalized as d_s/D_p depends on

$$\frac{d_s}{D_p} = \varphi\left(F_0, \frac{h_0}{D_p}, \frac{h}{D_p}, Re_f, \Delta, \frac{d_{50}}{D_p}\right), \tag{2}$$

where F_0 is the densimetric Froude number equal to $U_0/(gd_{50}\Delta)^{0.5}$, Re_f is the jet Reynolds number equal to U_0D_p/ν, and Δ is the relative submerged grain density equal to $(\rho_s - \rho)/\rho$.

A lot of studies were carried out in order to develop predictive formulas for d_s. They were obtained through empirical approaches traditionally based on dimensional analysis and data fitting to derive functional relations between non-dimensional groups.

FIG. 1. Photograph of an equilibrium scour produced by a propeller in a laboratory flume.

In unconfined conditions, by considering several operating propeller configurations (i.e., different values of U_0 and h_0) with sand or fine gravel beds, Hamill[8] suggested that the normalized maximum scour depth d_s/h_0 can be expressed as a function of the densimetric Froude number F_0 and h_0/D_p as follows:

$$\frac{d_s}{h_0} = 0.0467\left(\frac{F_0}{h_0/D_p}\right)^{1.39}. \tag{3}$$

The validity ranges of this formula are $5.55 \le F_0 \le 18.72$ and $0.81 \le h_0/D_p \le 2.87$.

Hong *et al.*[3] analyzed the longitudinal profiles of the unconfined scour hole and deposition mound measured in correspondence of the propeller axis by investigating several combinations of U_0, h_0, D_p, and d_{50}. They also examined different values of h, by demonstrating that the propeller scouring process was not affected by h when this latter was big enough (e.g., $h/D_p = 2.15$ to 6). They provided an empirical formula for the estimation of the maximum scour depth, which can be expressed as follows:

$$\frac{d_s}{D_p} = 0.265\left[F_0 - 4.114\left(\frac{h_0}{D_p}\right)\right]^{0.955}\left(\frac{h_0}{D_p}\right)^{-0.022}. \tag{4}$$

The formula proposed by Hong *et al.*[3] is valid for $6.08 \le F_0 \le 10.69$ and $0.5 \le h_0/D_p \le 1.5$. Using the same structure of Eq. (4) and adding their own data (that were collected keeping h constant and varying U_0, h_0, and d_{50}), Tan and Yüksel[4] provided the following new empirical formula:

$$\frac{d_s}{D_p} = 0.57\left[F_0 - 2.1\left(\frac{h_0}{D_p}\right)\right]^{0.33}\left(\frac{h_0}{D_p}\right)^{-1.1}, \tag{5}$$

which is valid for $1.89 \le F_0 \le 15.01$ and $0.77 \le h_0/D_p \le 3.08$.

Penna *et al.*[5] performed experimental tests in unconfined conditions: three different values of U_0 and three different values of h_0 were investigated. By demonstrating the asymmetry of the scour hole and deposition mound with respect to the propeller axis, Penna *et al.*[5] furnished a further equation for evaluating the absolute maximum scour depth (i.e., considering the whole study area and not only the scour profile in correspondence of the propeller axis), which is valid for $3.65 \le F_0 \le 9.54$ and $0.83 \le h_0/D_p \le 1.17$,

$$\frac{d_s}{D_p} = 0.06F_0^{1.34}. \tag{6}$$

Finally, Curulli *et al.*,[9] by varying U_0, h_0, d_{50}, and h, proposed the following equation for the unconfined scour depth:

$$\frac{d_s}{D_p} = 0.90F_{0p}^{1.30}\left(\frac{d_{50}}{D_p}\right)^{-0.05}\left(\frac{h_0}{D_p}\right)^{-0.53}, \tag{7}$$

where F_{0p} is the propeller Froude number, defined as $U_0/(gD_p)^{0.5}$ or, equivalently, as $F_0(d_{50}\Delta/D_p)^{0.5}$. The validity ranges of the formula are $0.40 \le F_{0p} \le 1.04$, $3.6 \times 10^{-3} \le d_{50}/D_p \le 8.4 \times 10^{-3}$, and $0.83 \le h_0/D_p \le 3.08$.

Table I shows a recap of the above-mentioned formulas.

All the literature works adopted an empirical methodology that requires combining dimensional analysis with a data-fitting technique. Unfortunately, many limitations related to this approach can be found: (i) the scouring phenomenon is regulated by a large number of variables (or dimensionless groups), whose effects are hard to recognize and isolate in the experiments; (ii) empirically developed formulas, which do not have a clear physical origin, are often restricted to the experimental data they have been derived from; (iii) these equations are usually a function of U_0 and, therefore, their structure mostly depends on the method used for its evaluation; and (iv) the empirical equations are often modified or recalibrated when additional datasets are considered, not providing a definitive model to assess the maximum scour depth induced by a propeller jet flow.

Recently, for different applications, e.g., the scour induced by a hydrokinetic turbine,[10] formed below a drop structure,[11,12] and at a bridge pier,[13,14] elegant theoretical formulations, based on phenomenological arguments, were proposed and validated.

Thus, the aims of this work are (1) to extend the phenomenological theoretical framework[10–14] for scouring produced by the propeller jet flow to provide a new model able to assess the maximum equilibrium scour depth under a range of propeller working conditions and (2) to confirm the proposed scour model using previously published datasets. As regards point (1), the theoretical formula proposed here does not require the preventive evaluation of U_0 (unlike the various empirical propeller jet formulas), because it directly relates the maximum scour depth to the propeller rotational speed (i.e., the number of revolutions per unit of time, n), which is known as input.

The paper is organized as follows: Sec. II provides the theoretical derivation of the scouring predictive formula; in Sec. III experimental data taken from the literature are used to validate the theoretical framework; Sec. IV combines the results obtained from Sec. III to derive a design formula for the maximum scour depth assessment; Sec. V is devoted to the discussion and the conclusions.

II. THEORETICAL MODEL

We start from the theory originally derived for fully rough turbulence in open-channel and pipe flow by Gioia and Bombardelli[15] and Gioia and Chakraborty.[16] The model relates the shear stress acting at the bottom of the scour surface in the proximity of the sediments exposed to the flow (i.e., the turbulent shear stress) to the characteristic length scales of the flow eddies (see inset in Fig. 2). The turbulent shear stress is defined as $\tau = -\rho\overline{u'w'}$ in the hypothesis of fully developed turbulence, where u' and w' are the velocity fluctuations in the longitudinal and wall-normal directions, respectively, and the over-line denotes the ensemble averaging operator. Following the approach proposed by Gioia and Bombardelli,[15] the fluctuations w' are dominated

TABLE I. Recap of the literature formulas considered in the present study.

Authors	Equations	Validity ranges
Hamill[8]	$\frac{d_s}{h_0} = 0.0467\left[\frac{F_0}{(h_0/D_p)}\right]^{1.39}$	$5.55 \le F_0 \le 18.72$ $0.81 \le \frac{h_0}{D_p} \le 2.87$
Hong *et al.*[3]	$\frac{d_s}{D_p} = 0.265\left[F_0 - 4.114\left(\frac{h_0}{D_p}\right)\right]^{0.955}\left(\frac{h_0}{D_p}\right)^{-0.022}$	$6.08 \le F_0 \le 10.69$ $0.5 \le \frac{h_0}{D_p} \le 1.5$
Tan and Yüksel[4]	$\frac{d_s}{D_p} = 0.57\left[F_0 - 2.1\left(\frac{h_0}{D_p}\right)\right]^{0.33}\left(\frac{h_0}{D_p}\right)^{-1.1}$	$1.89 \le F_0 \le 15.01$ $0.77 \le \frac{h_0}{D_p} \le 3.08$
Penna *et al.*[5]	$\frac{d_s}{D_p} = 0.06F_0^{1.34}$	$3.65 \le F_0 \le 9.54$ $0.83 \le \frac{h_0}{D_p} \le 1.17$
Curulli *et al.*[9]	$\frac{d_s}{D_p} = 0.90F_{0p}^{1.30}\left(\frac{d_{50}}{D_p}\right)^{-0.05}\left(\frac{h_0}{D_p}\right)^{-0.53}$	$0.40 \le F_{0p} \le 1.04$ $3.6 \times 10^{-3} \le \frac{d_{50}}{D_p} \le 8.4 \times 10^{-3}$ $0.83 \le \frac{h_0}{D_p} \le 3.08$

by eddies with the same size of the sediment roughness, characterized here by sediments of diameter d. In fact, the eddies with this size (i.e., the eddies pertaining in the inertial range $\eta \ll d \ll S$, where η is the Kolmogorov length scale, and S is the size of the energy-containing eddies[11]) are the most energetic in the whole length scale range of turbulent eddies, which can set among adjacent sediments and possibly move them. Conversely, the fluctuations u' scale with the velocity of the energy-containing eddies of the flow, U. Thus, the relative velocity scales are U and w_d for the length scales S and d, respectively. Hence, the wall shear stress scales as

$$\tau = -\rho\overline{u'w'} \sim \rho w_d U, \tag{8}$$

where the $\sim$ symbol means "scales as."

Equation (8) is applicable in the proximity of the wall (i.e., in the roughness sublayer,[15,16] and near the scour hole surface for different erosion phenomena[10–14]) and, hence, as presented here, in the vicinity

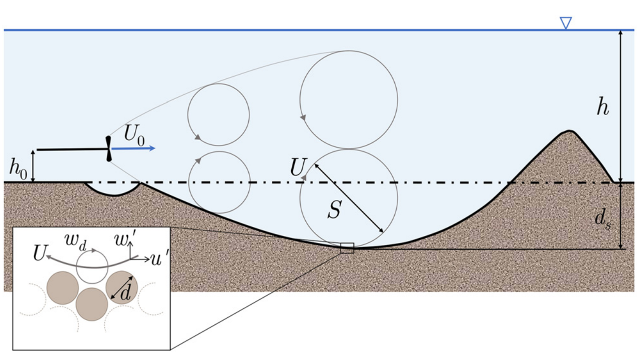

FIG. 2. Sketch of the theoretical scenario. The inset shows the characteristic velocity scales within the scour hole at the maximum scour depth.

of the bed scour produced by a propeller jet. Adopting the reasonings of Gioia and Bombardelli[11] and considering the specificity of the scour mechanism (Fig. 2), we assume that at the equilibrium conditions the largest eddies have a characteristic size comparable to the sum of the maximum scour depth, d_s, and the propeller gap, h_0, i.e., $S \sim d_s + h_0$.

As reported by Gioia and Chakraborty,[16] the phenomenological theory of turbulence is based on the assumption of equilibrium between the local dissipation rate and the production of the turbulent kinetic energy (TKE), and on two paradigms of turbulence:[17] (i) the turbulent energy production occurs at the largest eddy scales, and (ii) the rate of production (or dissipation) is independent of the viscosity. From these paradigms, it is possible to find a scaling expression for the rate of production (or dissipation) of turbulent energy per unit mass, ε, in terms of the velocity and of the size of the largest eddies. The result is the well-known Taylor scaling,[18] $\varepsilon \sim U^3/(d_s + h_0) \sim w_d^3/d$, leading to

$$w_d \sim U\left(\frac{d}{d_s + h_0}\right)^{1/3}, \tag{9}$$

which is one of the results of the Kolmogorov theory.[17,18] Precisely, the Kolmogorov derivation is applicable if turbulence, at small scales, has a homogeneous and isotropic behavior. Near the sediment, the turbulent field may not show these characteristics, owing to strain rates of flow. Nevertheless, the current knowledge indicates that Kolmogorov's theory is still valid for some anisotropic and inhomogeneous flows.[19,20] In addition, Gioia and Bombardelli[15] and Gioia and Chakraborty[16] demonstrated that using Kolmogorov's scaling close to rough and smooth walls, where isotropy and homogeneity are lost, allows finding many important empirical relations concerning hydraulics. Consequently, the hypothesis of the Kolmogorov scaling to theorize turbulence at small scale within a scour hole represents a plausible assumption.

Assembling Eqs. (8) and (9), the shear stress can be estimated as

$$\tau \sim \rho U^2 \left(\frac{d}{d_s + h_0}\right)^{1/3}. \tag{10}$$

Assuming, in the scour zone, a bulk characterization for the TKE dissipation rate per unit mass, $\varepsilon = P/M$, and hypothesizing that the largest eddies are mainly responsible for such a decay, we can define the power P as the work of a thrust force F_t induced by the propeller jet flow against U_0, and M as the mass of water within the largest eddies [i.e., $\sim\rho(d_s + h_0)^3$; see Fig. 2], leading to

$$\varepsilon = \frac{P}{M} \sim \frac{F_t U_0}{\rho (d_s + h_0)^3}. \tag{11}$$

Coupling $\varepsilon \sim U^3/(d_s + h_0)$ with Eq. (11), the velocity scale U can now assume the following relation:

$$U \sim \left[\frac{F_t U_0}{\rho (d_s + h_0)^2}\right]^{1/3}, \tag{12}$$

and, considering Eq. (10),

$$\tau \sim \left[\frac{\rho F_t^2 U_0^2 d}{(d_s + h_0)^5}\right]^{1/3}. \tag{13}$$

At the equilibrium state, the bottom scour shear stress τ reaches the critical shear stress for the inception of sediment motion τ_c (Ref. 21). Hence, at the equilibrium, $\tau = \tau_c$ with $\tau_c \sim (\rho_s - \rho) g d$, according to the Shields theory. Thus, from Eq. (13), it is possible to obtain the following expression:

$$d_s + h_0 \sim \left[\frac{\rho F_t^2 U_0^2}{(\rho_s - \rho)^3 g^3 d^2}\right]^{1/5}. \tag{14}$$

Based on the axial momentum theory, Stewart[22] expressed F_t as $\rho C_t D_p^2 (v_f^2 - v_a^2)$, where C_t is the thrust coefficient, v_f represents the fluid velocity, and v_a is the ship advance speed. Under bollard pull condition (i.e., $v_a = 0$), $v_f \sim U_0$ and $F_t \sim \rho C_t D_p^2 U_0^2$.

Additionally, considering the propeller as an actuator disk and applying the axial momentum theory, the efflux velocity is defined as $U_0 = n D_p (8C_t/\pi)^{1/2}$, where U_0 is expressed in m/s, n in rounds per second (rps), and D_p in m (Ref. 23). Therefore, $U_0 = n D_p (8C_t/\pi)^{1/2} \sim n D_p C_t^{1/2}$. Replacing F_t and U_0 in Eq. (14), one can derive the following relationship:

$$d_s + h_0 \sim \frac{n^{6/5} C_t D_p^2}{\Delta^{3/5} g^{3/5} d^{2/5}}. \tag{15}$$

Considering Eq. (15), it can be noted that, if $n = 0$ rps, $d_s + h_0 \sim 0$; since, $h_0 > 0$, it derives that $d_s \neq 0$, but this is not physically possible and, consequently, h_0 must be removed from Eq. (15). The same result can be obtained considering $S \sim d_s$ (Refs. 10 and 13) in Eq. (9). Additionally, owing to the fact that proportional relations are employed in the definition of the bottom shear stress, the thrust force, and the Shields critical stress, a multiplicative correction factor K must be introduced into the scaling relations [Eq. (15)], leading to the following equation:

$$d_s = K \frac{n^{6/5} C_t D_p^2}{\Delta^{3/5} g^{3/5} d^{2/5}}, \tag{16}$$

where K must be estimated empirically.

III. VALIDATION OF THE THEORETICAL MODEL

In order to test Eq. (16) by using experimental datasets, it is necessary to explain its validity conditions. This is done using the same considerations argued by Manes and Brocchini.[13] In particular, Eq. (16) was obtained under the strong hypothesis that the sediment size is comparable to the eddies with a length scale pertaining to the inertial range, i.e., $\eta \ll d \ll S$. As reported above, at the equilibrium state, the range of validity becomes $\eta \ll d \ll (d_s + h_0)$. To find the validity restriction of the predictive theory, it is necessary to replace $d_s + h_0$ (in which d_s is not known *a priori*) with a known parameter that is of the same order of magnitude (or minor to be more rigorous) as d_s. It is possible to verify that, from the literature datasets reported in Table II, the propeller diameter D_p is always less than $d_s + h_0$. Hence, $\eta \ll d \ll D_p$ is a stricter condition than the previous one. Assuming the largest eddy size equal to D_p and following the Manes and Brocchini[13] arguments, the corresponding scale velocity is U_0, the TKE dissipation can be computed as $\varepsilon \sim U_0^3/D_p$ and, hence, the bulk Kolmogorov length scale results

$$\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \sim \left(\frac{\nu^3 D_p}{U_0^3}\right)^{1/4}. \tag{17}$$

Consequently, Eq. (16) is valid if

$$\left(\frac{\nu^3 D_p}{U_0^3}\right)^{1/4} \ll d \ll D_p \tag{18}$$

or, equivalently, if

$$1 \ll \frac{D_p}{d} \ll Re_f^{3/4}. \tag{19}$$

TABLE II. Experimental range of the used datasets.

Dataset	n (rpm)	h_0 (m)	h (m)	D_p (m)	d_{50} (mm)	Δ	C_t
Hong *et al.*[3]	200–700	0.100–0.210	0.45–0.60	0.100–0.210	0.24–0.34	1.65	0.45
Tan and Yüksel[4]	590–745	0.100–0.200	0.50	0.065–0.130	0.52–8.30	1.61–1.72	0.61
Penna *et al.*[5]	270–480	0.068–0.096	0.25	0.082	0.69	1.66	0.35
Curulli *et al.*[9]	480–700	0.068–0.096	0.15–0.35	0.082	0.30–0.69	1.66–1.70	0.35

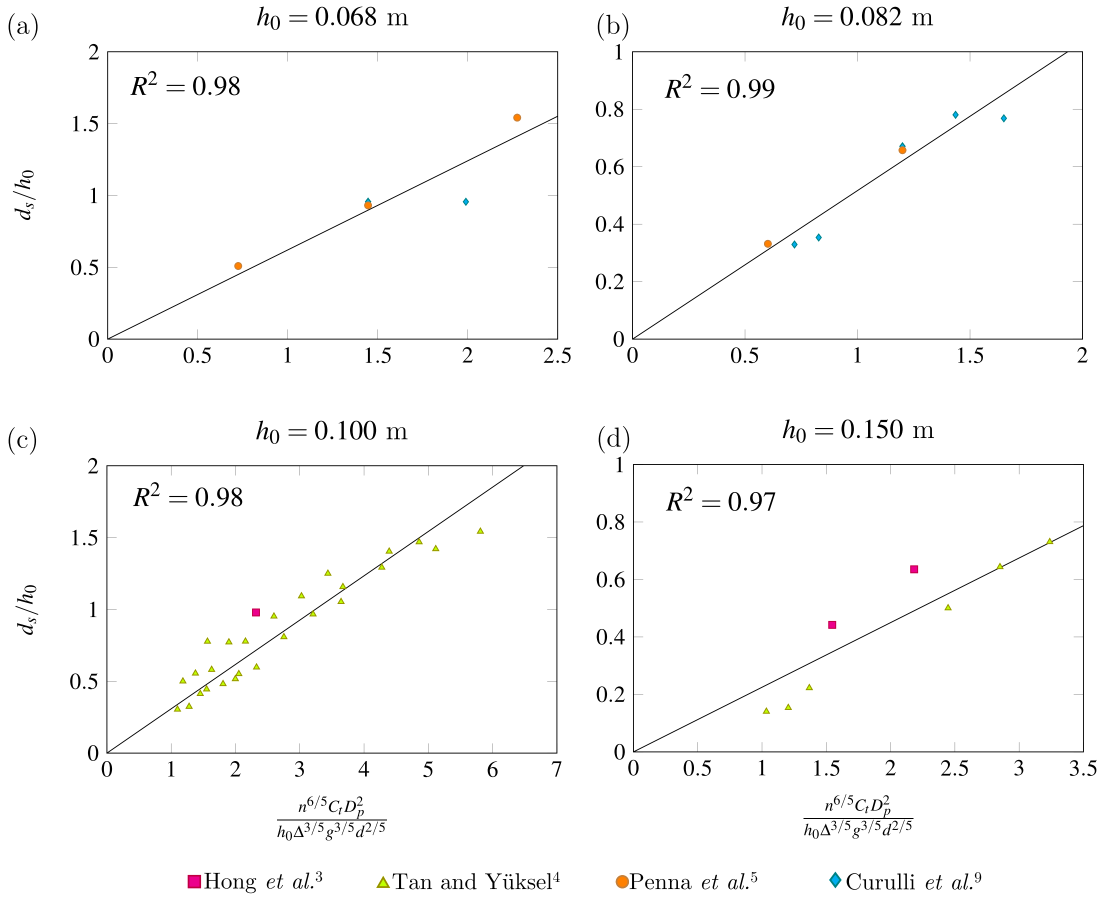

FIG. 3. Dimensionless equilibrium maximum scour depth d_s/h_0 as a function of $n^{6/5}C_tD_p^2/(h_0\Delta^{3/5}g^{3/5}d^{2/5})$ for different propeller gaps, h_0.

In addition, from geometrical consideration (see Fig. 2), Eq. (16) was derived in the case of $h + d_s > 2(h_0 + d_s)$; hence,

$$h > 2h_0 + d_s. \tag{20}$$

The validity of Eq. (16) is now verified against the experimental dataset by Hong *et al.*,[3] Tan and Yüksel,[4] Penna *et al.*,[5] and Curulli *et al.*[9] Considering Eqs. (19) and (20), the present proposed theory is tested with 58 experimental runs. Table II provides the most significant experimental characteristics of the aforementioned datasets.

Primarily, it is possible to note that the parameter h_0, which is mainly responsible for the scour evolution according to Hong *et al.*[3] and Tan and Yüksel,[4] does not appear in Eq. (16). Indeed, looking at Fig. 3, it is evident that the equilibrium maximum scour depth d_s is a function of $n^{6/5}C_tD_p^2/(\Delta^{3/5}g^{3/5}d^{2/5})$ for different values of propeller gap h_0. Thus, a linear regression can be applied to $n^{6/5}D_p^2/(\Delta^{3/5}g^{3/5}d^{2/5})$ (i.e., the independent variable) and d_s (i.e., the dependent variable), for each value of h_0. This allows the estimation of the corrective factor K as the slopes of the obtained linear laws.

Therefore, given the aforementioned datasets and the computed K values for each propeller gap, a power law regression (Fig. 4) can be performed. Practically, the following simple expression of K as a function of h_0 (expressed in m) can be determined:

$$K = \left(\frac{0.047}{h_0}\right)^{4/3}. \tag{21}$$

IV. A FINAL DESIGN FORMULA FOR THE MAXIMUM SCOUR DEPTH ASSESSMENT

The d_s assessment can be obtained by substituting Eq. (21) into Eq. (16), which leads to the following expression:

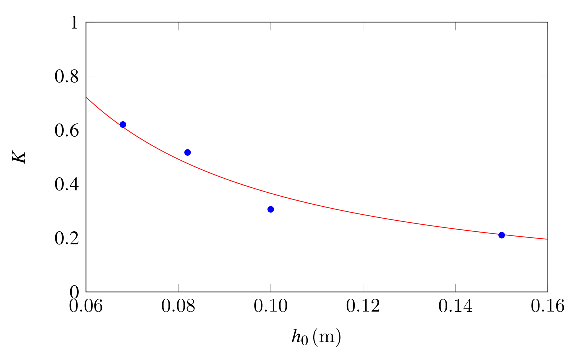

FIG. 4. Coefficient K as a function of the propeller gap h_0.

$$d_s = \left(\frac{0.047}{h_0}\right)^{4/3} \frac{n^{6/5} C_t D_p^2}{\Delta^{3/5} g^{3/5} d^{2/5}}. \quad (22)$$

The performance of Eq. (22) is verified against predictions made with other literature formulas. Specifically, Fig. 5 shows the agreement among the measured, d_{s_m}, and computed, d_{s_c}, maximum equilibrium scour depths using either the original or the literature formulas.[3–5,8,9] Figure 5(a) clearly demonstrates that the new methodology provides a much better data-collapse on the perfect agreement line, leading to a mean relative error $E_m = 19\%$ (computed as $|d_{s_c} - d_{s_m}|/d_{s_m} \times 100$). In addition, Figs. 5(b)–5(f) report the correlation between the measured and computed maximum equilibrium scour depth using the formulas for the scour prediction suggested by Hamill[8] ($E_m = 33\%$), Hong *et al.*[3] ($E_m = 89\%$), Tan and Yüksel[4] ($E_m = 33\%$), Penna *et al.*[5] ($E_m = 66\%$), and Curulli *et al.*[9] ($E_m = 83\%$), respectively.

Thus, the previous equation has the advantage of being applicable over a broader spectrum of experimental conditions than the existing literature formulas, leading to a lower mean relative error. In fact, when the literature equations are used outside their applicability ranges, they return high errors in the prediction of the maximum scour depth. Note that the use of new datasets may improve the estimation of the correction factor K, without altering the structure of Eq. (16).

In order to recommend a final design formula for the equilibrium scour depth assessment, an approach similar to that of Melville and Coleman[24] is proposed herein. In fact, Melville and Coleman[24] recommended a design equation for estimating the maximum scour depth at

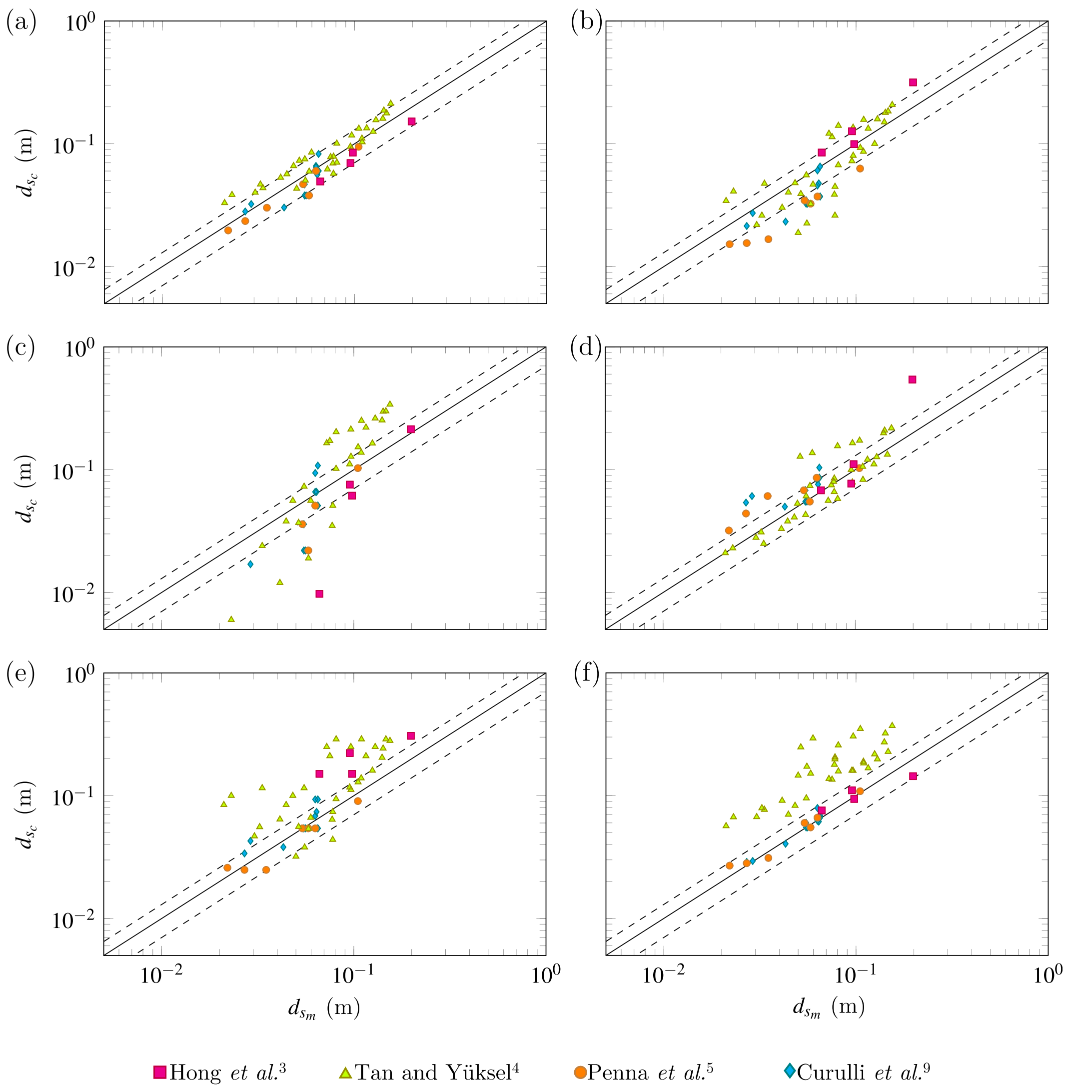

FIG. 5. Computed, d_{s_c}, vs measured, d_{s_m}, maximum depths at equilibrium: (a) d_{s_c} from the proposed theory; (b) d_{s_c} from the Hamill[8] equation (first row of Table I); (c) d_{s_c} from the Hong *et al.*[3] equation (second row of Table I); (d) d_{s_c} from the Tan and Yüksel[4] equation (third row of Table I); (e) d_{s_c} from the Penna *et al.*[5] equation (fourth row of Table I); and (f) d_{s_c} from Curulli *et al.*[9] equation (fifth row of Table I). The solid lines are the perfect agreement lines between computed and experimental measured values, whereas the dashed lines are the relative ±30% error bounds.

piers based on fitting curves that overlap the data plots. As can be seen in Fig. 5(a), the +30% error dashed line practically quasi-overlap all the data. Hence, a design formula with a security factor equal to 30% of the physically based Eq. (22) can be used. The final formula is, consequently, the following:

$$d_s = \left(\frac{0.057}{h_0}\right)^{4/3} \frac{n^{6/5} C_t D_p^2}{\Delta^{3/5} g^{3/5} d^{2/5}}, \tag{23}$$

being $1.3 \times (0.047)^{4/3} \approx (0.057)^{4/3}$, in which h_0 is expressed in m.

V. CONCLUSIONS

The current study presents a theoretical model to compute the maximum equilibrium scour depth produced by propeller jets positioned in aquatic environments characterized by an erodible sediment bed. The analytical formulation develops on theoretical works,[10–14] addressing different scour problems with the phenomenological theory of turbulence introduced by Gioia and Bombardelli[15] and Gioia and Chakraborty.[16]

Specifically, the scouring development is dominated by geometry-specific vortices of turbulence that regulate themselves to dissipate TKE up to the sediment dimension scale, at a rate characterized by the power dissipated through the thrust (drag) force exerted by the propeller (structure) itself. We hypothesized that the dissipation process induced by a propeller jet in the near-bed region can be explained using a conceptual scenario.

Hence, a theoretical model was proposed and validated in order to obtain a new formula for local-scour prediction. First, the performance of the theoretical equation was analyzed against existing laboratory data and demonstrated to be considerably more reliable than that given by empirical literature formulas[3–5,8,9] when these latter are used outside their applicability ranges. Second, in order to recommend a formula for design purposes, the proposed equation was further corrected with a security factor equal to 1.3.

In future works, we plan to perform an experimental campaign using non-uniform bed sediments in order to include the grain size distribution effect in a new more complete predictive design formula.

ACKNOWLEDGMENTS

This research was funded by the Italian "*PAC Calabria 2014–2020 - Asse Prioritario 12, Azione B) 10.5.12*" and "*Programma Operativo Regione (POR) Calabria FESR-FSE 2014/2020*."

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Francesco Coscarella: Conceptualization (lead); Data curation (lead); Formal analysis (lead); Funding acquisition (supporting); Investigation (equal); Methodology (lead); Project administration (supporting); Resources (supporting); Software (equal); Supervision (equal); Validation (equal); Visualization (lead); Writing – original draft (lead); Writing – review & editing (equal). **Giuseppe Curulli:** Conceptualization (equal); Data curation (lead); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Resources (supporting); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Nadia Penna:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (supporting); Writing – review & editing (equal). **Roberto Gaudio:** Conceptualization (supporting); Data curation (supporting); Formal analysis (supporting); Funding acquisition (lead); Investigation (equal); Methodology (equal); Project administration (lead); Resources (lead); Software (equal); Supervision (lead); Validation (equal); Visualization (equal); Writing – original draft (supporting); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

[1]J. Carlton, *Marine Propellers and Propulsion* (Butterworth-Heinemann, 2018).
[2]G. Hamill, H. Johnston, and D. Stewart, "Propeller wash scour near quay walls," J. Waterw., Port, Coastal, Ocean Eng. **125**, 170–175 (1999).
[3]J.-H. Hong, Y.-M. Chiew, and N.-S. Cheng, "Scour caused by a propeller jet," J. Hydraul. Eng. **139**, 1003–1012 (2013).
[4]R. İ. Tan and Y. Yüksel, "Seabed scour induced by a propeller jet," Ocean Eng. **160**, 132–142 (2018).
[5]N. Penna, F. D'Alessandro, R. Gaudio, and G. R. Tomasicchio, "Three-dimensional analysis of local scouring induced by a rotating ship propeller," Ocean Eng. **188**, 106294 (2019).
[6]R. Ettema, B. W. Melville, and G. Constantinescu, *Evaluation of Bridge Scour Research: Pier Scour Processes and Predictions* (The National Academies Press, Washington, DC, 2011).
[7]Y. Yüksel, R. İ. Tan, and Y. Celikoglu, "Determining propeller scour near a quay wall," Ocean Eng. **188**, 106331 (2019).
[8]G. A. Hamill, "Characteristics of the screw wash of a manoeuvring ship and the resulting bed scour," Ph.D. thesis (Queen's University of Belfast, 1987).
[9]G. Curulli, N. Penna, and R. Gaudio, "Improved formulation for the geometric characteristics of the scour hole and of the deposition mound caused by a rotating propeller jet on a mobile bed," Ocean Eng. **267**, 113175 (2023).
[10]M. Musa, M. Heisel, and M. Guala, "Predictive model for local scour downstream of hydrokinetic turbines in erodible channels," Phys. Rev. Fluids **3**, 024606 (2018).
[11]G. Gioia and F. A. Bombardelli, "Localized turbulent flows on scouring granular beds," Phys. Rev. Lett. **95**, 014501 (2005).
[12]F. A. Bombardelli and G. Gioia, "Scouring of granular beds by jet-driven axisymmetric turbulent cauldrons," Phys. Fluids **18**, 088101 (2006).
[13]C. Manes and M. Brocchini, "Local scour around structures and the phenomenology of turbulence," J. Fluid Mech. **779**, 309–324 (2015).
[14]F. Coscarella, R. Gaudio, and C. Manes, "Near-bed eddy scales and clear-water local scouring around vertical cylinders," J. Hydraul. Res. **58**, 968–981 (2020).
[15]G. Gioia and F. Bombardelli, "Scaling and similarity in rough channel flows," Phys. Rev. Lett. **88**, 014501 (2001).
[16]G. Gioia and P. Chakraborty, "Turbulent friction in rough pipes and the energy spectrum of the phenomenological theory," Phys. Rev. Lett. **96**, 044502 (2006).
[17]A. N. Kolmogorov, "The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers," C. R. Acad. Sci. URSS **30**, 301–305 (1941).
[18]U. Frisch, *Turbulence: The Legacy of an Kolmogorov* (Cambridge University Press, 1995).
[19]B. Knight and L. Sirovich, "Kolmogorov inertial range for inhomogeneous turbulent flows," Phys. Rev. Lett. **65**, 1356 (1990).
[20]R. D. Moser, "Kolmogorov inertial range spectra for inhomogeneous turbulence," Phys. Fluids **6**, 794–801 (1994).

[21]A. Shields, "Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," Ph.D. thesis (Technical University Berlin 1936).
[22]D. P. J. Stewart, "Characteristics of a ship's screw wash and the influence of quay wall proximity," Ph.D. thesis (Queen's University of Belfast, 1992).
[23]M. Fuehrer and K. Römisch, "Effects of modern ship traffic on islands and ocean waterways and their structures," in *24th International Navigation Congress, Leningrad* (1977).
[24]B. W. Melville and S. E. Coleman, *Bridge Scour* (Water Resources Publication, 2000).