An energy-efficient green-vehicle routing problem with mixed vehicle fleet, partial battery recharging and time windows

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Abstract

We investigate a specific version of the Green Vehicle Routing Problem, in which we assume the availability of a mixed vehicle fleet composed of electrical and conventional (internal combustion engine) vehicles. These are typically light- and medium-duty vehicles. We allow partial battery recharging at any of the available stations. In addition, we use a comprehensive energy consumption model which can take into account speed, acceleration, deceleration, load cargo and gradients. We propose a matheuristic embedded within a large neighborhood search scheme. In a numerical study we evaluate the performance of the proposed approach.

Keywords: logistics; green vehicle routing; mixed fleet; matheuristic; hybrid large neighborhood search.

1 Introduction

The planning and management of freight logistics systems have traditionally aimed at improving transportation efficiency in terms of cost, time and profit. More recently, we have witnessed a growing interest in the environmental aspects of transportation, such as pollution, noise and congestion. In this context, developing environmentally-friendly and efficient transport and distribution systems, in order to ensure the best trade-off between cost minimization and negative environmental externalities reduction, represents an important challenge.

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We study the problem of managing electric vehicles (EVs) and conventional vehicles with the aim of reducing the costs derived from the routing and the recharging operations. We propose an energy consumption model in which several realistic aspects are considered and where acceleration and deceleration are taken into account. The idea of considering the energy produced by the braking system was suggested in the context of the electrical vehicle traveling salesman problem by Bay and Limbourg [3] at an Odysseus Conference in 2015. Here we extend for the first time this idea to a vehicle routing problem with a mixed fleet, composed of electrical and conventional diesel vehicles having different capacities. We also consider the possibility of partially recharging the EVs at any available recharging station. Combining these elements distinguished our problem from the previous contributions and offers an increased degree of realism.

We propose a mathematical formulation and we design and implement a matheuristic, which combines the resolution of the proposed model, embedded within the large neighborhood search scheme.

1.1 State of the art

Here, we briefly review the most interesting scientific contributions in green logistics. For a complete survey the reader is referred to Lin et al. [21]. We can distinguish between two important categories of problems: Pollution Routing Problems (PRPs) and Green Vehicle Routing Problems (GVRPs). The former problems aim at minimizing pollution, in particular carbon emissions. The latter make use of alternative fuel vehicles (AFVs) and alternative fuel stations (AFSs), and the main objective is to minimize energy consumption in transportation.

PRP. Bektas and Laporte [4] introduced and modeled the PRP. In this problem the amount of pollution is evaluated by an energy-based model, which takes into account of load, among other factors. They explicitly considered the effect of CO_2 emissions, showed the difficulty of solving the PRP to optimality, and mentioned the possibility of several extensions. They proposed a non-linear mixed integer programming formulation to mathematically represent the problem, whose objective is to minimize the cost of greenhouse gas (GHG) emissions, the operational costs of drivers and fuel consumption. This work was extended by Demir et al. [6] who considered several vehicle speeds and proposed and tested an Adaptive Large Neighbourhood Search (ALNS) algorithm. Jabali et al. [15] focused on the Time-Dependent VRP. They presented a model that considers travel time, fuel and CO_2 emissions costs, and proposed a tabu search procedure to solve the problem. Franceschetti et al. [11] studied the Time-Dependent PRP, a PRP extension that takes traffic congestion into account. The authors proposed an integer linear programming formulation and considered a special case called the departure time and speed optimization problem. Tajik et al. [30] investigated the time window pickup-delivery PRP. In this PRP variant, pickup and delivery operations are considered and vehicle speed is stochastic. The authors solved a mixed integer linear programming model (MILP) and introduced a robust variant. Koç et al. [17] studied a scientific extension that considers a heterogeneous vehicle fleet. Table 1 summarizes the main contributions on the PRP.

Table 1: Summary of the PRP contributions in the scientific literature

		PRP				
Reference	Algorithm	Time windows	Time dependent	Pickup and delivery	Uncertain data	Heterogeneous fleet
Bektaş and Laporte (2011)		√				
Demir et al. (2012)	Heuristic	√				
Jabali et al. (2012)	Heuristic		√			
Franceschetti et al. (2013)		√	√			
Tajik et al. (2014)		√		√	√	
Koç et al. (2014)	Heuristic	√				√

G-VRP. One of the first articles on the G-VRP is that of Kara et al. [16]. In this work the authors consider a capacitated VRP and propose a linear integer formulation in order to reduce energy consumption. Gonçalves et al. [13] investigated the VRP with pickups and deliveries. They analyzed three different scenarios. The first one is an application of the VRP with pickups and deliveries with a conventional fleet; in the second one the fleet is composed of conventional vehicles and electrical uncapacitated vehicles; in the last one they considered only capacitated EVs. The authors proposed a MILP model and applied a p-median algorithm in order to decompose the original set of customers. The problem was then solved on each cluster.

Erdoğan and Miller-Hooks [8] presented a MILP formulation for the G-VRP. Moreover, they proposed several techniques in order to compute a solution that minimizes the total distance traveled, while incorporating stops for the refuelling of AFVs at AFSs. Vehicles are assumed to be uncapacitated and time window constraints are not taken into account. Customer time windows, demands and capacity constraints were considered by Schneider et al. [26] who focused on the Electrical VRP with time windows (VRPTW) with Recharging Stations (E-VRPTW). Recharging vehicles at any of the available stations is allowed, but the batteries are always fully recharged. These authors presented a MILP formulation and proposed a hierarchical objective function of the E-VRPTW. The first objective is the minimization of the number of vehicles; the second one is the minimization of the total traveled distance. Their approach is a metaheuristic that combines variable neighbourhood search (VNS) and tabu search. Felipe et al. [9] described the G-VRP with Multiple Technologies and Partial Recharge. Partial battery recharges and overnight depot charging are allowed. The recharging operations can be performed with different technologies, each of them having a different recharging time and cost. They proposed a constructive algorithm based on a greedy generation method, a deterministic local search and a simulated annealing.

Ćirović et al. [5] investigated the G-VRP with a heterogeneous fleet composed of environmental friendly and unfriendly vehicles. However, when defining a route, friendly and unfriendly vehicles are considered separately. The authors used a neuro-fuzzy model to formulate the problem under study. Goeke and Schneider [12] considered a mixed fleet of conventional vehicles and EVs. The authors formulated the E-VRP with time windows and mixed fleet, in which the EVs can be charged at the available charging stations (CSs). Charging times vary according to the battery level when the EV arrives at the CS and charging is always done up to maximum battery capacity. They proposed a comprehensive energy consumption model which considers speed, vehicle mass and gradient. Desaulniers et al. [7] presented four variants of the E-VRPTW. In the first one, batteries must be fully charged and at most one recharge per route is allowed; in the second one multiple recharges are allowed, in the third one only one partial battery recharging per route is allowed, in the last one multiple and partial battery recharges are allowed. The authors developed two branch-and-price-and-cut algorithms for these problems. Hiermann et al. [14] introduced the electric fleet size and mix vehicle routing problem with time windows and recharging stations. They considered a heterogeneous fleet of EVs in which each vehicle is characterized by its fixed cost, battery and load capacity, energy consumption and charging rate. Each vehicle can be fully charged at a CS.

Koç and Karaoglan [18] developed a simulated annealing heuristic based on an exact solution approach to solve the G-VRP introduced by Erdoğan and Miller-Hooks [8]. In their formulation, the authors used new decision variables in order to allow multiple visits to the CSs without augmenting the networks with dummy nodes. Based on this work, Leggieri and Haouari [19], proposed a new formulation for the E-VRPTW. In order to assess the effectiveness of their approach, the authors solved their model with CPLEX and compared the results with those obtained by the branch-and-cut algorithm of Koç and Karaoglan [18].

Since the installation and operation costs of the network highly impact company's strategies, several authors introduced decisions about location and technology of CSs in the E-VRPs. Thus Yang and Sun [32] considered the electric vehicles battery swap stations location routing problem whose aim is to determine the locations of battery swap stations, as well as the routing plan of EVs. Li-ying and Yuan-bin [20] focused on the EV multiple charging station location-routing problem with time windows. Schiffer and Walther [24] introduced the electric location routing problem with time windows and partial recharging in which the EVs can be charged at any node in the network. Schiffer and Walther [25] introduced the location-routing problem with intra-route facilities which focuses on determining the location of facilities for intermediate stops. The facilities are not depots and do not necessarily coincide with customers. Intra-route facilities allow for intermediate stops on a route in order to keep the vehicle operational. Paz et al. [23] defined the multi-depot electric vehicle location-routing problem with time windows and a homogeneous fleet of EVs. They considered the possibility of recharging the EV at the CSs or to swap the battery at a battery swap station. The goal is to determine the number and location of CSs and depots, as well as the number of EVs and their routes. Recently, Macrina et al. [22] proposed a local search heuristic for a green mixed fleet (EVs and internal combustion commercial vehicles (ICCVs)) vehicle routing problem with partial battery recharging and time windows. The authors showed how time windows features and partial recharges of battery instead of full recharges may influence solution quality.

The mathematical formulation proposed in our paper can be viewed as an extension of the model presented by Erdoğan and Miller-Hooks [8], which is the first routing model that considers recharging stations. However significant modifications have been introduced in order to represent the specific characteristics of the problem under study. Schneider et al. [26] and Felipe et al. [9] have already extended the model presented in [8]. In the first contribution only complete recharges are allowed, while in the second one the batteries can be partially recharged with different technologies. However, in both papers, it is assumed that the fleet is exclusively composed of EVs. Here we extend the model in order to also handle conventional vehicles. Gonçalves et al. [13] and Goeke and Schneider [12] considered a mixed fleet. However, in both contributions, the batteries must be fully recharged. In addition, in the first of these papers, it is assumed that the EVs are uncapacitated. Goeke and Schneider [12] proposed a comprehensive energy consumption model. However, they did not consider the effects of the acceleration phase and of the braking process on the energy consumption. Ćirović et al. [5] considered a fleet exclusively made up of conventional or electrical vehicles. In the problem studied in our paper, we simultaneously take these two possibilities into account. Table 2 summarizes the main contributions on the G-VRP.

1.2 Aim and organization of this paper

From this literature review, it is clear that scarce attention has been devoted to the use of a mixed vehicle fleet. Most studies assume the energy consumption proportional to traveled distance, and consider partial battery recharging when a fleet is composed of only AFVs. Nobody has combined the four features considered in our paper, namely mixed

Table 2: Summary of the G-VRP contributions in the scientific literature

Reference	Algorithm	Model	Time windows	Fixed charging	Partial recharge	Multiple technologies	Linear charging	Energy consumption linear to distance	Energy consumption model	Pickup and delivery	Mixed Fleet (EVs and ICCVs)
Gonçalves et al. (2011)		√		√				√		√	√
Conrad and Figliozzi (2011)	Heuristic	√	√				√	√			
Erdoğan and Miller-Hooks (2012)	Heuristics	√		√				√			
Schneider et al. (2014)	Heuristic	√	√				√	√			
Felipe et al. (2014)	Heuristic	√			√	√	√	√			
Ćirović et al. (2014)	Heuristic										√
Goeke and Schneider (2015)	Heuristic	√	√				√		√		√
Desaulniers et al. (2016)	Exact	√	√		√		√	√			
Macrina et al. (2018)	Heuristic	√	√		√	√	√	√			√
This work	Heuristic	√	√		√	√	√		√		√

fleet, partial battery recharging, time windows and comprehensive energy consumption model with braking energy generation. In particular, we model a comprehensive energy consumption function which can take into account speed, acceleration, deceleration, loaded cargo and road gradients. We consider the effects of the acceleration and braking phases on energy consumption, as well as other realistic features related to the life span of the battery. Indeed, full recharges can damage the battery and the last 10% of recharge requires considerable time. Hence we also need to constrain the state of charge of the battery.

The remainder of this paper is structured as follows. In Section 2, we highlight the main characteristics of the problem under study and describe the mathematical model developed for its representation. In particular we use two energy consumption models for the conventional and electric vehicles described in Sections 2.1 and 2.2, respectively. In Section 3 we describe the algorithm we have developed to solve the problem. In Section 4 we present the computational experiments and the numerical results. In Section 5 we summarize the conclusions.

2 The energy-efficient green mixed fleet vehicle routing problem with partial battery recharging and time windows

We formulate our problem as follows. Let $\mathcal{N}$ be the set of customers, and let $\mathcal{R}$ be the set of recharging stations. We will also need copies of recharging stations to account for multiple visits at the same stations. Thus, let $\mathcal{R}'$ be the set of all stations and their copies, i.e., $R \subset R'$. Let $\mathcal{V} = \mathcal{R} \cup \mathcal{N}$ and $\mathcal{V}' = \mathcal{R}' \cup \mathcal{N}$. The problem will be defined on the graph $G(V', A)$, where $\mathcal{A}$ is the set of arcs.

The depot 0 is a particular element belonging to the set $\mathcal{R}'$, that is the recharging station where vehicle routes start and its dummy copy $0'$ is the node where the routes end. Each customer $i \in \mathcal{N}$ has a demand q_i (in kg) and a service time s_i (in hours). All customers must be visited by a single vehicle. Each node $i \in \mathcal{V}'$ has a time window $[e_i, l_i]$, where e_i and l_i represent the earliest and latest times at which service may start at node i, respectively. For each arc $(i, j) \in \mathcal{A}$, d_{ij} denotes the distance from i to j [km], while t_{ij} the travel time from i to j [hours]. We impose a limit T on the maximum total duration of a route [hours], that is, the end of the time window associated with the depot node is set equal to T.

A heterogeneous fleet of vehicles, composed of n^E EVs and n^C ICCVs, is available. The two types of vehicles (electrical and conventional) are characterized by different capacities, denoted as Q^E_{max} and Q^C_{max} [kg] for the electrical and conventional vehicles, respectively, and different curb weight denoted by w^E and w^C respectively. Furthermore, for each electrical vehicle let B^E denote the maximum battery capacity [kWh], while for each conventional vehicle B^C is the fuel tank maximum capacity [L]. The recharging cost [€/kWh] is equal to ω.

Each recharging station $i \in \mathcal{R}'$ has a charging mode, which can be slow, moderate or fast, and is therefore characterized by a recharging speed ρ_i [kWh per hour]. Partial battery recharging is allowed at any recharging station.

2.1 The fuel consumption for the conventional vehicles

We develop an energy consumption model following the ideas presented in [4].

Let u be the transported amount of cargo [kg], we calculate the mechanical power, that is, the power at the wheels, as follows:

$$p^M(u) = [(a(t) + g\text{sin}\theta + gC_r\text{cos}\theta)(u + w) + (0.5C_dA\rho)v(t)^2]v(t), \quad [kW] \tag{1}$$

where $a(t) = dv(t)/dt$ represents the acceleration [m/s^2] and takes negative values when the vehicle decelerates, while g denotes the gravitational constant (9.81 m/s^2). The angle of the road is θ, here assumed to be zero, C_r is the coefficient of rolling resistance, C_d is the drag coefficient, A is the frontal surface Area [m^2], ρ is the air density [kg/m^3)], w denotes the curb weight of the vehicle [kg], and $v(t)$ is the speed [m/s]. As in [4] we suppose that the engine power demand associated with running engine losses and the operation of vehicle accessories such as air conditioning, are zero. Thus $p^M(u)$ can be viewed as the second-by-second engine power output.

We use the emission model of Barth et al. [2] and Barth and Boriboonsomsin [1], applied to the PRP by Bektas and Laporte [4], Demir et. al [6], Koç et. al [17] to estimate fuel consumption, in order to convert the mechanical power into fuel consumption.

Since this consumption model does not use the deceleration, recently Suzuki and Lan (2018) [29] extended it, by including negative values of acceleration to study the fuel consumption of trucks in congested areas. They ensured that the fuel use rate cannot be lower than that of engine idling, to avoid the unreal case of negative fuel burn during the deceleration phase.

Let $v(t)$ be the vehicle speed traversing an arc of length d. We can calculate the fuel consumption [L] on this arc as

$$F(v(t)) = (\xi/\kappa\Psi)(kN_eD_e + p^M(u)/\eta_e\eta_{dt})d/v(t), \quad [L] \tag{2}$$

where ξ is the fuel-to-air mass ratio, k is the engine friction factor [kJ/rev/L], N_e is the engine speed [rev/s], D_e is the engine displacement [L], η_e and η_{dt} are the efficiency parameters for diesel engines and the drive train efficiency respectively, while κ is the heating value of a typical diesel fuel [kJ/g], and Ψ is a conversion factor from [g/s] to [L/s].

In order to represent the relationship between distance and travel time, we follow the idea presented in [31]. In this work the authors study the vehicle motion and performance in urban passenger transportation and they propose a model based on several cases that can occur by varying the acceleration during a trip. In the spirit of what proposed in [31], we suppose that each arc is composed of three phases ($h = 1, 2, 3$) associated with different values of acceleration a and speed levels v. We consider a first phase, the "acceleration phase", in which the value of a is positive, the second one is the "constant speed phase" where the value of a is zero and the speed is constant, finally, the third phase is the "deceleration phase" in which the value of a is negative.

Let v_{max} be the maximum speed reachable, and t_{ij} depends on whether a vehicle can reach its maximum speed. Let d^c_{ij} be the distance required for a vehicle to reach v_{max}, also called critical distance. We can distinguish two main cases: in the first one $d_{ij} < d^c_{ij}$, in the second one $d_{ij} \geq d^c_{ij}$ (see figure 1).

Case 1 (figure 1(a)). In Figure 1(a), since $d_{ij} < d^c_{ij}$, the travel time for the arc and the distance d_{ij} will be marked by a prime ($'$), as well as the maximum speed as v', where $v' < v_{max}$. In this case we have only two states, i.e., the acceleration and deceleration phases associated with the indices $h = 1$ and $h = 3$, respectively .

Case 2 (figure 1(b)). If $d_{ij} \geq d^c_{ij}$, the vehicles reaches v_{max} and maintains this speed until it decelerates. Thus, we have the three phases on arc (i, j).

In our three-phase model, we define a start time t_{ij_h} and a speed $v(t_{ij_h})$, associated with each phase and arc (i, j). In particular, t_{ij_h} represents the time at which the vehicle enters state h on arc (i, j), and $v(t_{ij_h})$ is the speed of the vehicle at the beginning of state h.

Then, starting from (1) we can write the mechanical power consumed to traverse the part of arc (i, j) associated to the phase h as

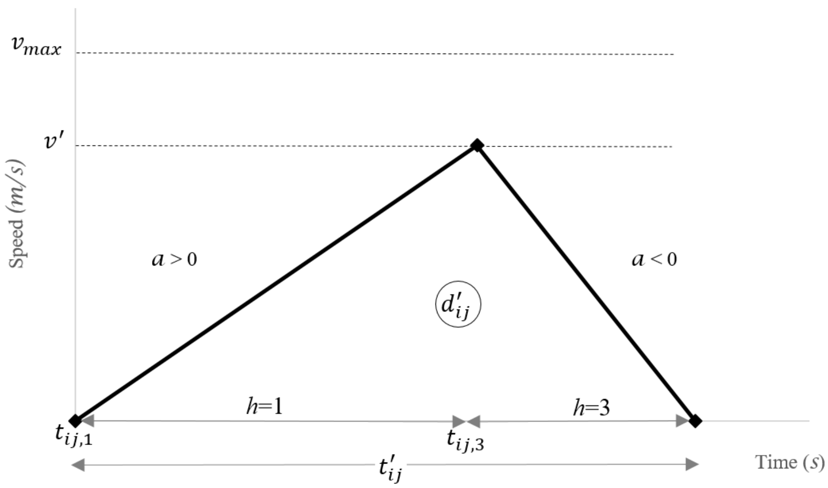
$$p^M_{ij_h}(u_i) = [(a(t_{ij_h}) + g\sin\theta + gC_r\cos\theta)(u_i + w) + (0.5C_dA\rho)v(t_{ij_h})^2]v(t_{ij_h}). \ [kW] \tag{3}$$

Hence, the mechanical power consumed to travel on the arc (i, j) is calculated as

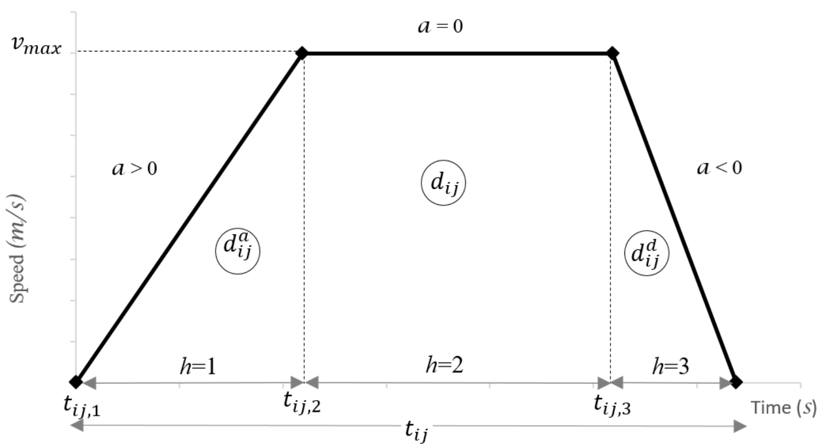
$$p^M_{ij}(u_i) = \sum_{h=1,2,3} p^M_{ij_h}(u_i). \ [kW] \tag{4}$$

Using (2) and denoting the time necessary to travel the distance d_h, related to the phase h, as $\bar{t}_h$, we write the fuel consumption as

$$f_{ij}(u_i) = \sum_{h=1,2,3} (\xi/\kappa\Psi)(kN_eD_e + p^M_{ij_h}(u_i)/\eta_e\eta_{dt})\bar{t}_h. \ [L] \tag{5}$$



(a) Case 1: v_{max} is not reached



(b) Case 2: v_{max} is reached

Figure 1: Two cases of travel regimes on the arc (i, j)

As mentioned before, to avoid the estimation of negative fuel burn during deceleration phase, we consider only the case of extremely gentle deceleration and we guarantee that the fuel use rate is not lower than that of engine idling.

2.2 The energy consumption for the electric vehicles

To define the energy consumption for the EVs, we have adapted the model of Fiori et. al. [10], which is a simple EV energy model that computes the instantaneous energy consumption and braking energy regeneration for the EVs using a second-by-second vehicle speed, acceleration and roadway grade as input variables. We calculate the power of an electric motor starting from the mechanical power defined in (1), and considering η the energy efficiency from battery-to-wheels. When the vehicle is in traction mode, the energy flows from the motor to the wheels and the power of electric motor is higher than the power at the wheels, thus the power at the wheels is assumed to be positive. In contrast, when the energy flows from wheels to the motor, the power at the electric motor is lower than the power at the wheels and the latter is assumed to be negative. Thus, η is given by η^+ in motor mode, i.e., the energy is positive and represents the discharged electric energy, and by η^- in recuperating mode, i.e., the energy is negative and represents the recuperated electric energy. It is possible to calculate the energy consumption $p^E(u)$, for an arc of length d, starting from equation (1) as

$$p^E(u) = (p^M(u)/\eta)t, \ \ [kWh] \tag{6}$$

where t is the time necessary to travel the distance d, and

$$\eta = \begin{cases} \eta^+ \leq 1, & \text{if } p^E(u) \text{ is positive, and } 0 \leq p^M(u) \leq 100 \text{ kW} \\ \eta^- \geq 1, & \text{if } p^E(u) \text{ is negative, and } -100 \leq p^M(u) \leq 0 \text{ kW} \end{cases}. \tag{7}$$

Using our three-phase model to define the mechanical power $p^M_{ij_h}(u_j)$ in (3), it is possible to calculate the energy consumption $p^E_{ij_h}(u_j)$ [KW] necessary to travel the distance d_h related to phase h of the arc (i, j). We denote the travel time as $\bar{t}_h$:

$$p^E_{ij_h}(u_i) = ([(a(t_{ij_h}) + g\text{sin}\theta + gC_r\text{cos}\theta)(u_i + w) + (0.5C_dA\rho)v(t_{ij_h})^2]v(t_{ij_h})/\eta)\bar{t}_h, \ \ [kWh]. \tag{8}$$

It is worth observing that $\eta = \eta^+$ when $h = 1, 2$ and $\eta = \eta^-$ when $h = 3$. Therefore, the energy p^E_{ij} [KW] consumed to travel on the arc (i, j) is defined as

$$p^E_{ij}(u_i) = \sum_{h=1,2,3} p^E_{ij_h}(u_i). \ \ [kWh] \tag{9}$$

2.3 The mathematical model

In order to model the problem we define the decision variables as follows:

$$x^E_{ij} = \begin{cases} 1, & \text{the electrical vehicle travels from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \quad (i, j) \in \mathcal{A}$$

$$x^C_{ij} = \begin{cases} 1, & \text{the conventional vehicle travels from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \quad (i, j) \in \mathcal{A}$$

u_i^C amount of load left in the conventional vehicle after visiting node i [kg], $i \in \mathcal{V}'$
u_i^E amount of load left in the electric vehicle after visiting node i [kg], $i \in \mathcal{V}'$
z_j^E amount of energy available when arriving at node j [kWh], $j \in \mathcal{V}'$
z_j^C amount of fuel available when arriving at node j [L], $j \in \mathcal{V}'$
$z_{i0'}^{depotE}$ amount of energy available when arriving at depot $0'$ from node i [kWh], $i \in \mathcal{V}'$
$z_{i0'}^{depotC}$ amount of fuel available when arriving at depot $0'$ from node i [L], $i \in \mathcal{V}'$
g_{ij} amount of energy recharged by the electrical vehicle at the node i for travelling to j [kWh], $i \in \mathcal{R}, j \in \mathcal{V}'$
$p_{ij_h}^E(u_i^E)$ amount of energy necessary to travel from i to j transporting the cargo (u_i^E) ; $i, j \in \mathcal{V}'$, $h = 1, 2, 3$
τ_j arrival time of the vehicle to the node j [h], $j \in \mathcal{V}'$

The Mixed Integer Linear Program that models our problem is as follows:

$$\text{Minimize} \quad \sum_{i \in R'} \sum_{j \in V'} \omega_i^g g_{ij} + \omega^e \sum_{i \in V' \setminus \{0'\}} (B^E - z_{i0'}^{depotE}) + \omega^f \sum_{(i,j) \in A} f_{ij}(u_i^C) + \sum_{(i,j) \in \mathcal{A}} cd_{ij}(x_{ij}^E + x_{ij}^C) \tag{10}$$

subject to

$$\sum_{j \in \mathcal{V}'} (x_{ij}^E + x_{ij}^C) = 1, \qquad i \in \mathcal{N} \tag{11}$$

$$\sum_{j \in \mathcal{V}'} x_{ij}^E \leq 1, \qquad i \in \mathcal{R}' \tag{12}$$

$$\sum_{j \in \mathcal{V}' \setminus \{0\}} x_{ij}^E - \sum_{j \in \mathcal{V}' \setminus \{0'\}} x_{ji}^E = 0, \qquad i \in \mathcal{V}' \tag{13}$$

$$\sum_{j \in \mathcal{V} \setminus \{0\}} x_{ij}^C - \sum_{j \in \mathcal{V} \setminus \{0'\}} x_{ji}^C = 0, \qquad i \in \mathcal{V} \tag{14}$$

$$\sum_{j \in \mathcal{V}'} x_{0j}^E \leq n^E \tag{15}$$

$$\sum_{j \in \mathcal{V}} x_{0j}^C \leq n^C \tag{16}$$

$$\tau_j \geq \tau_i + (t_{ij} + s_i) x_{ij}^E - M(1 - x_{ij}^E), \qquad i \in \mathcal{N}, j \in \mathcal{V}' \tag{17}$$

$$\tau_j \geq \tau_i + (t_{ij} + s_i) x_{ij}^C - M(1 - x_{ij}^C), \qquad i \in \mathcal{V}, j \in \mathcal{V} \tag{18}$$

$$\tau_j \geq \tau_i + t_{ij} x_{ij}^E + \frac{1}{\rho_i} g_{ij} - M(1 - x_{ij}^E), \qquad i \in \mathcal{R}', j \in \mathcal{V}' \tag{19}$$

$$e_j \leq \tau_j \leq l_j, \qquad j \in \mathcal{V}' \tag{20}$$

$$u_j^E \geq u_i^E + q_j x_{ij}^E - Q_{max}^E (1 - x_{ij}^E), \qquad i \in \mathcal{V}' \setminus \{0, 0'\}, j \in \mathcal{V}' \setminus \{0\} \tag{21}$$

$$u_j^C \geq u_i^C + q_j x_{ij}^C - Q_{max}^C (1 - x_{ij}^C), \qquad i \in \mathcal{V} \setminus \{0, 0'\}, j \in \mathcal{V} \setminus \{0\} \tag{22}$$

$$u_{0'}^C \leq Q_{max}^C \tag{23}$$

$$u_{0'}^E \leq Q_{max}^E \tag{24}$$

$$u_0^C = 0 \tag{25}$$

$$u_0^E = 0 \tag{26}$$

$$z_j^E \leq z_i^E - \sum_{h=1,2,3} p_{ij_h}^E(u_i^E) + B^E(1 - x_{ij}^E), \qquad i, j \in V' \setminus \{0, 0'\} \tag{27}$$

$$z_j^E \leq z_i^E + g_{ij} - \sum_{h=1,2,3} p_{ij_h}^E(u_i^E) + B^E(1 - x_{ij}^E), \qquad i \in R', j \in V' \setminus \{0, 0'\} \tag{28}$$

$$z_0^E = 0.9B \tag{29}$$

$$z_{i0'}^{depotE} \leq z_i^E - \sum_{h=1,2,3} p_{i0'_h}^E(u_i^E)) + B^E(1 - x_{i0'}^E), \qquad i \in V' \setminus \{0, 0'\} \tag{30}$$

$$z_{i0'}^{depotE} \leq z_i^E + g_{i0'} - \sum_{h=1,2,3} p_{i0'_h}^E(u_i^E) + B^E(1 - x_{i0'}^E), \qquad i \in R' \tag{31}$$

$$0 \leq z_{i0'}^{depotE} \leq 0.9 B^E x_{i0'}^E, \qquad i \in V' \setminus \{0'\} \tag{32}$$

$$0.1 B \leq z_j^E \leq 0.9 B^E, \qquad j \in V' \setminus \{0\} \tag{33}$$

$$g_{ij} \leq 0.9B - z_i^E + B^E(1 - x_{ij}^E), \qquad i \in R', j \in V' \tag{34}$$

$$p_{ij_h}^E(u_i^E) \geq \tag{35}$$

$$([(a(t_{ij_h}) + g\sin\theta + gC_r\cos\theta)(u_j + w) + (0.5 C_d A \rho) v(t_{ij_h})^2] v(t_{ij_h}) \bar{t}_h / \eta^+)$$

$$-B^E(1 - x_{ij}^E), \qquad i, j \in V', h = 1, 2$$

$$p_{ij_h}^E(u_i^E) \geq \tag{36}$$

$$([(a(t_{ij_h}) + g\sin\theta + gC_r\cos\theta)(u_j + w) + (0.5 C_d A \rho) v(t_{ij_h})^2] v(t_{ij_h}) \bar{t}_h / \eta^-)$$

$$-B^E(1 - x_{ij}^E), \qquad i, j \in V', h = 3$$

$$z_j^C \leq z_i^C - f_{ij}(u_i^C) + B^C(1 - x_{ij}^C), \qquad i, j \in V \setminus \{0'\} \tag{37}$$

$$0 \leq z_j^C \leq B^C, \qquad j \in V \setminus \{0\} \tag{38}$$

$$z_0^C = B^C \tag{39}$$

$$z_{i0'}^{depotC} \leq z_i^C - f_{i0'}(u_i^C) + B^C(1 - x_{i0'}^C), \qquad i \in V \setminus \{0'\} \tag{40}$$

$$0 \leq z_{i0'}^{depotC} \leq 0.9 B^C x_{i0'}^C, \qquad i \in V \setminus \{0'\} \tag{41}$$

$$x_{ij}^E, x_{ij}^C \in \{0, 1\}, i \in \mathcal{V}', j \in \mathcal{V}'; u_i^E, u_i^C, \tau_i \geq 0, i \in \mathcal{V}',$$

$$g_{ij} \geq 0, i \in \mathcal{R}', j \in \mathcal{V}'. \tag{42}$$

Objective. The objective function is the sum of four terms. The first one, that is $\sum_{i \in R'} \sum_{j \in V'} \omega_i^g g_{ij}$, is the cost of the energy recharged during the route. In particular ω_i^g is the unit cost of recharge [€/kW] at station i and it depends of the available technology at station i. The second one ($\omega^e \sum_{i \in V' \setminus \{0'\}} (B^E - z_{i0'}^{depotE})$) is the cost of the energy recharged to the depot, the unit cost of energy is ω^e [€/L]. The third one, that is $\omega^f \sum_{(i,j) \in A} f_{ij}(u_i^C)$ is the fuel cost, with ω^f the unit cost of fuel [€/L]. The last one, $\sum_{(i,j) \in \mathcal{A}} c d_{ij}(x_{ij}^E + x_{ij}^C)$ is the travel cost, where c is the cost [€/km] which depends on the traveled distance.

VRP constraints. The constraints (11) ensure that each customer is visited exactly once, whereas conditions (12) impose that each recharging station can be visited at most once. Constraints (13) and (14) are the flow conservations constraints, whereas conditions (15) and (16) ensures that the total number of used vehicles (electrical and conventional, respectively) is less than the available ones.

Time windows constraints. Constraints (17)–(19) define the variables τ, which represent the arrival time at node j, where j can be a customer served by either a classical or an electrical vehicle, or it can be a recharge station. Time windows constraints are represented by conditions (20).

Capacity constraints. Conditions (21)–(26) represent the capacity constraints, for the electrical and the conventional vehicles.

Electrical energy constraints. Constraints (27) and (28) define the variables z^E ensuring that the capacity of the electric vehicles battery is not exceeded, in particular after visiting a customer and a recharge station, respectively. Constraint (29) ensures that the vehicle is fully charged at the starting node, considering that the full charge can damage the battery, while constraints (30) and (31) define the amount of energy available when the vehicles arrive at ending node. Contraints (32) and (33) define the state of charge of battery, since we consider that recharging the last 10% of battery requires long times, we allow a complete discharge only to the depot. Constraints (34) are used to represent the partial battery recharging. Constraints (35) and (36) linearize the energy consumption as described in Section 2.2.

Classical fuel tank constraints. Constraints (37) set the fuel level equal to the maximum fuel tank capacity reduced by the fuel necessary to traverse the arc, constraints (38) and (39) restrict the fuel level, while constraints (40) and (41) define the available amount of fuel and restrict the fuel level for the ending node respectively.

Finally, conditions (42) define the domains of variables.

3 A matheuristic algorithm

We have developed a matheuristic to solve our problem. In particular, we propose a hybrid version of large neighborhood search (HLNS) algorithm introduced by Shaw [27], which iteratively removes and inserts customers from the routes in the solution. We generated an initial feasible solution Γ_{current} by solving the proposed model with CPLEX. We fixed a limit $\bar{t}$ on the execution time, to find an initial feasible solution. We randomly applied removal and insertion operators, which remove and insert customers and CSs, obtaining the solutions Γ_{remove} and Γ_{insert} respectively.

The procedure was repeated with the best solution found Γ_{best} or an accepted current solution whose cost is minor than cost (Γ_{best}) υ where υ is a tolerance input parameter, until the stopping criteria (i.e. a maximum number of iterations $k^{\max}$) was met.

3.1 Removal and Insertion operators

We now describe our removal and insertion operators. Removal operators remove ζ customers and then place them in a removal list. The value of ζ is selected from an interval $[\zeta^-, \zeta^+]$, where ζ^- and ζ^+ are input parameters. Insertion operators insert ζ customers in the destroyed solution by following several rules. We introduce a temporary tabu status which forbids the insertion of customers in routes which have been recently removed from routes, as well as the removal of customers which have been recently inserted in routes.

Algorithm 1 . Hybrid large neighborhood search (HLNS)

```
Generate the initial solution Γ_current
Γ_current → Γ_best
while k < k^max do
  Apply a removal operator to Γ_current and obtain Γ_remove
  Apply a insertion operator to Γ_remove and obtain Γ_insert
  Γ_insert → Γ_temporary
  if cost(Γ_temporary) < cost(Γ_best) then
    Γ_temporary → Γ_best
    Γ_temporary → Γ_current
    k = 0
  else if cost(Γ_temporary) < cost(Γ_best) υ) then
    Γ_temporary → Γ_current
    k ← k + 1
  else
    k ← k + 1
  end if
end while
return best solution Γ_current
```

3.1.1 Removal operators

Our HLNS uses the following four destroy operators:

1. **Random removal**: iteratively removes ζ customers from a solution.
2. **Worst distance removal**: iteratively removes the unfavorable customers. The operator sorts all the customers in descending order of cost, where the cost is the sum of distances of the customer from the preceding and succeeding nodes in the route.
3. **Worst time removal**: similar to the worst distance removal, the operator sorts all the customers in descending order of cost, where cost for a node i is calculated as $|\tau_i - e_i|$.
4. **Route removal**: this operator randomly selects a route and remove it from the solution.

3.1.2 Insertion operators

We use four insertion operators.

- **Greedy customers insertion**: iteratively determines the best feasible insertion position for a customer by calculating the insertion cost between two nodes in the route.

- **Greedy customers and CS insertion**: iteratively determines the best insertion position for a customer, which satisfies time windows and capacity constraints, by calculating the insertion cost between two nodes in the route. If the battery capacity constraint is violated, a CS is inserted in the best feasible position before visiting the customers.

- **Greedy new route insertion**: this operator initializes a new route, electric or conventional by evaluating the insertion cost of a customer between the starting and ending nodes.

- **GRASP insertion**: this operator sorts customers in a list of size L in ascending order of insertion cost. Then it selects the next customer to be inserted among the best feasible r^{GRASP} insertions, where r^{GRASP} is a random number in $[0, r^{GRASP}L/2]$.

4 Computational study

This section presents the results of our computational experiments. Our instances are inspired from the scientific literature (see [28] and [26]). We solved the model with CPLEX 12.5, by imposing a time limit of three hours. The HLNS was implemented in Java. All computations were performed on an Intel 2.60 GHz processor and 16 GB of RAM. Tables 3 summarizes the parameter setting used for our computational results.

The remainder of this section is organized as follows. In Section 4.1 we describe the generation of the instances. In Section 4.2 we investigate how considering acceleration and deceleration in the consumption energy model may influence the evaluation of the energy spent. In Section 4.3 we present our computational results to assess the behaviour of the HLNS algorithm.

4.1 Generation of instances

For each E-VRPTW instance, with customer locations defined by Cartesian coordinates (a_i,b_i), we generate the charging station in the square ($\min_i \{a_i\}$, $\min_i \{b_i\}$) and upper right hand corner ($\max_i \{a_i\}$, $\max_i \{b_i\}$) by solving a location problem.

We stress that the problem addressed refers to the routing of conventional and electric vehicles with fixed recharging station positions. Hence we have to define a meaningful displacement of the recharging stations.

Table 3: Setting of conventional and electrical vehicles parameters

Notation	Description	Value
g	Gravitational constant [m/s^2]	9.81
θ	Road angle	0
C_r	Coefficient of rolling resistance	0.01
C_d	Drag coefficient	0.7
A	Frontal surface Area [m^2]	3.912
ρ	Air density [kg/m^3]	1.225
w	Curb weight (kg/m^3]	6350
v	Speed [m/s]	13.88
ξ	Fuel-to-air mass ratio	1
k	Engine friction factor [kJ/rev/L]	0.2
N_e	Engine speed [rev/s]	33
D_e	Engine displacement [L]	5
η_e	Efficiency parameter for diesel engines	0.9
η_{dt}	Drive train efficiency	0.4
κ	Heating value of a typical diesel fuel [kJ/g]	44
Ψ	Conversion factor [g/L]	737
B^C	Fuel tank maximum capacity [kg])	3650
B^E	Maximum battery capacity [kWh]	80
ρ	Recharging speed [W/min]	0.0083
ω_i^g	Unit cost of recharge at station i [€/kW]	0.4
η^+	Energy efficiency in motor mode [12]	0.76
η^-	Energy efficiency in recuperating mode [12]	1.27
ω^e	Unit cost of energy [€/kW]	0.17
ω^f	Unit cost of fuel (€/L)	1.3
c	Driver wage [€/km]	0.195

Let $\mathcal{N}$ be the set of customers defined in Section 2 and let $\mathcal{Y}$ be the set of candidate charging stations. We define the decision variables as follows:

$$y_j = \begin{cases} 1, & \text{charging station } j \text{ is activated} \\ 0, & \text{otherwise} \end{cases} \quad j \in \mathcal{Y}$$

$$x_{ij} = \begin{cases} 1, & \text{customer } i \text{ is served by } j \\ 0, & \text{otherwise} \end{cases} \quad i \in \mathcal{N}, j \in \mathcal{Y}.$$

Thus we formulate and solve the following problem:

$$\begin{aligned}
\text{Minimize} \quad & c^p \sum_{i \in N} \sum_{j \in \mathcal{Y}} p_{ij} x_{ij} + c^f \sum_{j \in Y} y_j && (43) \\
\text{subject to} \quad & p_{ij} x_{ij} \leq B^E y_j, \quad i \in \mathcal{N}, j \in \mathcal{Y} && (44) \\
& p_{ij} x_{ij} \geq 1, \quad i \in \mathcal{N} && (45) \\
& \sum_{j \in \mathcal{Y}} y_j \geq H && (46) \\
& x_{ij} \in \{0,1\} \quad i \in \mathcal{N}, j \in \mathcal{Y} && (47) \\
& y_j \in \{0,1\} \quad j \in Y, && (48)
\end{aligned}$$

where H is a minimum number of charging stations that we want to activate. We calculate p_{ij} as described in Section 2.2. The objective function (43) minimize the cost of the energy consumed, where c^p is the unitary cost of energy [€/kWh], and c^f [€] is the activation cost for the charging station. Constraints (44) are the battery constraints, constraints (45) guarantee that all the customers are served, and constraints (46) state that at least H charging stations are activated. We fix a configuration by considering a partially laden vehicle in recuperating mode which travel on the arc with a constant speed. With this configuration, we ensure that each customer may be reached by using at least one charging station at minimum cost.

Once the location problem has been solved, since it is well known that the acceleration influences travel time and energy consumption (see Vuchic [31]), we have considered non-negative values for the acceleration and deceleration rates. To guarantee the comfort and safety of the rider and the integrity of the goods, we have fixed the values of acceleration in the range $[1.0, 1.8]$ $[m/s^2]$, and the value of deceleration in the range $[2.0, 3.0]$ $[m/s^2]$, arbitrarily on each arc, as suggested by Vuchic [31].

4.2 Effects of acceleration and deceleration on energy consumption

We have studied the impact of acceleration and deceleration, in particular during the departure and braking phases. Acceleration is a key factor in the determination of energy consumption for both ICCVs and EVs and allows the estimation of the regenerated electrical energy during the braking phase. Our experiments consist of solving to optimality a set of small-size instances, by considering the following three energy consumption models: the model in which the energy consumption is assumed to be proportional to the traveled distance, the energy consumption model proposed in Goeke and Schneider [12], where the acceleration and braking processes are not taken into account and the one proposed in this paper.

The solutions are compared in terms of the energy consumed, by considering only feasible configurations. Since the proportional model may underestimate the energy consumption, some configurations generated using this model may be infeasible for our model. More specifically, constraints (27) and (28) and (30)–(36) of the MILP model presented in Section 2.3 are not satisfied.

We first consider the optimal solutions obtained by solving the MILP proposed in Section 2.3. Table 4 presents the results. For each instance, we report its name, the energy consumed by using the linear function model (Model A), and that used by considering the energy consumption models. In particular, Model B neglects the acceleration while Model C considers acceleration and braking phases. We also report the percentage errors in consumed energy (Δ_A and Δ_B) calculated as (P^E_C- P^E_A)/P^E_C and (P^E_C- P^E_B)/P^E_C, respectively, where P^E_C is the energy consumption evaluated by using model C and P^E_A and P^E_B represents the energy consumptions for Models A and B, respectively.

Table 4 clearly shows that the energy consumption values calculated by using the proportional consumption model are gravely underestimated. Indeed, the percentage error is on average about 70%. This uncorrected evaluation of energy has a great impact on costs and feasibility of the solutions. It follows that the estimate of energy, given by the proportional consumption model, does not capture the real behavior of the EVs.

Of interest is also the comparison between Model B and Model C: on average the percentage error is about 4%.

In order to investigate in more detail the effect of considering the acceleration and braking phases in the energy consumption evaluation, we have calculated, using Model B and Model C, the energy consumed on each arc on a toy instance with two customers, shown in Figure 2. We have considered the three-phase model introduced in Section 2.2 and for each phase we have evaluated the acceleration values (see Figure 2). Obviously, since Model B does not take acceleration and deceleration into account, and assumes a constant speed on each arc, the energy consumption is always positive. Looking at the

Table 4: Comparison of energy consumption models

Test	Energy consumption [Wh]			Energy percentage error	
	Model A	Model B	Model C	Δ_A	Δ_B
MF_C101_5	1941.46	6229.49	6461.55	69.95%	3.59%
MF_C103_5	1689.20	5509.56	5732.92	70.54%	3.90%
MF_C206_5	1934.91	6915.41	7167.37	73.00%	3.52%
MF_C208_5	1503.42	5671.83	5902.16	74.53%	3.90%
MF_R104_5	1624.30	5014.72	5234.77	68.97%	4.20%
MF_R105_5	1652.19	5080.53	5310.81	68.89%	4.34%
MF_R202_5	1424.18	4761.13	4998.70	71.51%	4.75%
MF_R203_5	2069.09	6647.20	6886.05	69.95%	3.47%
MF_RC105_5	2439.28	7420.92	7638.12	68.06%	2.84%
MF_RC108_5	2547.13	7750.07	7972.58	68.05%	2.79%
MF_RC204_5	1962.84	6445.58	6676.76	70.60%	3.46%
MF_RC208_5	1882.31	6083.58	6302.34	70.13%	3.47%

tables in Figure 2, the difference in the energy consumption evaluation obtained with Model C is related to the acceleration and braking phases, i.e., when the value of a_h is not zero. On the one hand the amount of energy needed in the acceleration phase, is on average six times higher than the energy evaluated by assuming a constant speed. On the other hand, when we consider the braking phase, the energy values are negative because of the regenerating braking system.

4.3 Assessment of the HLNS

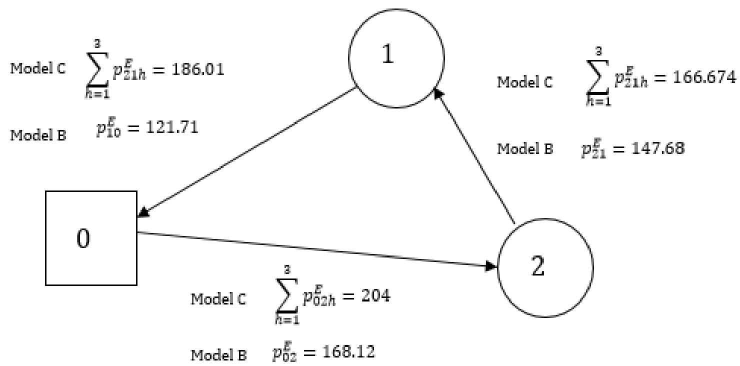
In order to assess the performance of our proposed HLNS, we first solve the problem using CPLEX, and we then compare the results with those obtained with the HLNS. We evaluate the performance of the proposed heuristic along two dimensions: solution quality and computational effort. We use the parameter setting presented in Table 5.

Table 5: Parameters setting for instances with 10 and 15 customers

	10 customers	15 customers
$k^{\max}$	5	10
$\bar{t}$ [seconds]	[10,20]	[10,50]
n^{C}	1	2
n^{E}	1	2

Tables 6 and 7 summarizes the results obtained on the instances with 10 and 15

Figure 2: Toy instance solution using Model B and Model C



Model C

$arc\ (i,j)$	(0,2)		(2,1)		(1,0)	
	a_h	p^E_{ijh}	a_h	p^E_{ijh}	a_h	p^E_{ijh}
$h=1$	1.77	**229.06**	1.23	**249.94**	109	**285.934**
$h=2$	0.00	**101.78**	0.00	**71.94**	0.00	**35.50**
$h=3$	−2.01	**-126.58**	−2.77	**-135.87**	−2.77	**-154.76**
$\sum_{h=1}^{3} p^E_{ijh}$	204.58		186.01		166.674	

Model B

$arc\ (i,j)$	(0,2)		(2,1)		(1,0)	
	a_h	p^E_{ijh}	a_h	p^E_{ijh}	a_h	p^E_{ijh}
$h=1$	0.00	**31.52**	0.00	**47.064**	0.00	**57.75**
$h=2$	0.00	**101.78**	0.00	**71.94**	0.00	**35.50**
$h=3$	0.00	**34.83**	0.00	**28.97**	0.00	**28.459**
$\sum_{h=1}^{3} p^E_{ijh}$	168.12		147.68		121.71	

customers, respectively. The first column shows the name of the instance, the second column g_{cost} represents the percentage gap in cost defined as $g_{cost} = (c^H - c^M)/c^M$, where c^H is the cost provided by the heuristic and c^M is the cost obtained solving the model. In the third column we report the speedup value, i.e., the ratio between the computational time required by CPLEX and that of the heuristic. CPLEX finds an optimal solution only on the 10-customer instances. The results presented in Tables 6 and 7 clearly demonstrate the advantage, in terms of efficiency, of the proposed matheuristic. Overall, the algorithm is less time consuming than CPLEX.

The HLNS is on average about 26.51 and 222.90 times faster than CPLEX for instances with 10 and 15 customers, respectively. It is worth observing that our matheuristic is also effective. Indeed, the gap on the cost for this class of instances is on average less than 3%. In particular, for those instances with 10 customers, the HLNS finds an optimal solution in three cases and the average on the cost gap is 1.06%. For the 15-customers instances, our matheuristic outperforms CPLEX. If finds the best solution for one instance (MF_R20215), and the average gap on the cost is 4.8%.

Table 6: Computational results for instances with 10 customers

Test	g_cost	Speedup
MF_C101_10	0.13%	31.08
MF_C104_10	0.03%	25.98
MF_C202_10	0.00%	67.91
MF_C205_10	0.00%	1.85
MF_R102_10	0.32%	43.32
MF_R103_10	0.11%	17.71
MF_R201_10	0.50%	45.60
MF_R203_10	0.00%	5.51
MF_RC102_10	4.31%	19.33
MF_RC108_10	2.26%	20.25
MF_RC201_10	3.13%	28.28
MF_RC205_10	1.90%	11.23
Average	1.06%	26.51

Table 7: Computational results for instances with 15 customers

Test	g_cost	Speedup
MF_C103_15	4.83%	221.60
MF_C106_15	7.67%	222.92
MF_C202_15	7.94%	223.10
MF_C208_15	0.00%	222.85
MF_R102_15	2.04%	223.54
MF_R105_15	8.01%	223.18
MF_R202_15	-2.53%	222.99
MF_R209_15	5.02%	222.74
MF_RC103_15	7.98%	223.10
MF_RC108_15	7.88%	223.16
MF_RC202_15	0.07%	223.21
MF_RC204_15	8.69%	222.79
Average	4.80%	222.90

Now we present the computational results on instances with 20, 25, 30 and 35 customers. We use the parameter setting presented in Table 8.

Table 8: Parameters setting for medium-sized instances

	20	25	30	35
$k^{\max}$	10	10	10	10
$\bar{t}$ [seconds]	[10,50]	[50,100]	[50,100]	[50,100]
n^{C}	4	5	5	5
n^{E}	4	5	5	5

Table 9 summarizes the results obtained for medium-size instances with 20, 25 and 30 customers. The first column displays the name of the instance, the second one the computational time [ms], the third one the total cost, the fourth and fifth columns show the number of conventional and electrical vehicles respectively. We also report, in the last line of each class of instances, the average for all statistics. Overall, the HLNS finds feasible solutions within a reasonable amount of time. Indeed, it solves instances with 20, 25 and 30 customers in about two minutes.

Since CPLEX is unable to find an initial solution for instances belonging to the class R and RC with up to 30 customers, Table 10 summarizes the results obtained for medium-size instances with 35 customers for instances belonging to the clustered class C. On average, our matheuristic solves this class of instances in about four minutes.

Table 9: Computational results for instances with 20,25 and 30 customers

Test	Time	Objective	# ICCV	# EV
MF_C101_20	50432	228.81	3	0
MF_C102_20	51307	216.88	2	1
MF_R101_20	50267	291.09	2	1
MF_R102_20	55266	308.65	3	0
MF_RC101_20	57568	388.39	3	2
MF_RC102_20	55326	387.44	3	2
Average	50495	303.54	2.67	1.00
MF_C101_25	50495	258.08	4	0
MF_C102_25	50326	251.56	4	0
MF_R101_25	50364	357.99	3	0
MF_R102_25	50348	393.80	3	0
MF_RC101_25	50469	568.24	4	1
MF_RC102_25	50360	602.47	5	1
Average	50394	405.36	3.83	0.33
MF_C101_30	60780	325.31	4	1
MF_C102_30	60500	349.43	5	0
MF_R101_30	60606	570.39	4	3
MF_R102_30	60626	435.22	3	3
MF_RC101_30	60551	625.07	2	5
MF_RC102_30	60591	581.12	2	2
Average	60609	481.08	3.33	2.33

Table 10: Computational results for instances with 35 customers

Test	Time	Objective	# ICCV	# EV
MF_C101_35	100830	494.19	5	1
MF_C102_35	100743	508.30	4	3
MF_C103_35	100896	427.56	5	1
MF_C104_35	100785	393.85	2	4
MF_C105_35	100726	491.05	5	1
Average	100796	462.99	4.2	2

5 Conclusions

We have investigated a new green vehicle routing problem variant. We have considered a mixed fleet composed of electrical and conventional vehicles, with time windows associated with each customer and partial battery recharging. We have proposed a comprehensive energy consumption model which can take into account several real-life parameters. In particular we have introduced new features related to the acceleration and braking phases, by incorporating the effect of the braking regenerating system in the model. We have defined a mixed integer program whose aim is to route the fleet of vehicles in order to serve all the customers satisfying the time window constraints, minimizing the transportation and the recharging costs. To highlight the importance of the proposed energy consumption model, we have shown that models in which energy consumption is proportional to distance and that do not consider the acceleration and deceleration may underestimate the energy spent, and hence yield infeasible solutions. However, a more conservative energy consumption model could reject some potential less expensive feasible routes. We have also developed a matheuristic based on large neighborhood search. In order to test the model and to assess the performance of our proposed heuristic, we have solved the model with CPLEX for small instances. Overall, the matheuristic is less time consuming than CPLEX. We have also shown that our HLNS can solve medium-size instances within a reasonable amount of time.

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